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Research Article

Rethinking the teaching of Euclidean geometry

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Euclidean geometry teaching and learning in South Africa has a unique history. It is one of the topics that are characterised by teaching-learning difficulties as demonstrated by learner underachievement related to the topic. There is death of research that explored teachers' explanatory talk during Euclidean geometry lessons. Thus, to address this research gap, within qualitative research approach, we employed non-structured classroom observations with 6 teachers, to explore and understand how they make Euclidean geometry concepts and principles available for the learners. We used Adler and Ronda's concept of explanatory talk to make sense of teachers' classroom practices. We used content analysis to understand what teachers said and write on the board during teaching. In this paper, we focus on one of the four teachers' lesson, to demystify how their explanatory talk or lack thereof made geometry concepts available for learners to learn. The findings reveal that the teacher used questions-and-answer strategy to engage learners in the lessons but did not provide explanations to the learners during the lesson, to guide them on the nature of the concepts and their relatedness.

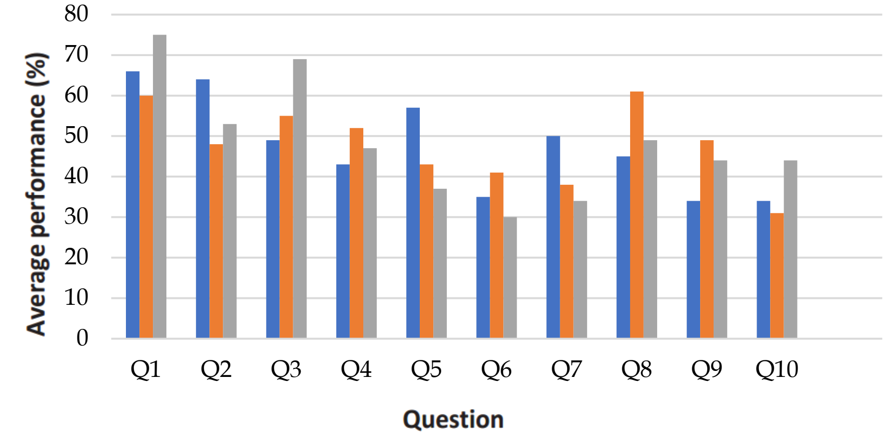
Keywords: Euclidean geometry, explanatory talk, teaching, teachers, observations

1. Introduction

One of the objectives of the National Curriculum Statement for Grades R–12 is to produce learners who can effectively communicate using visual, symbolic, or verbal skills in a variety of contexts (Department of Basic Education [DBE], 2011). This objective is in line with the value of geometry, particularly Euclidean geometry, which is envisaged to teach learners how to solve geometrical problems using a variety of representational strategies and logical thinking. Bansilal and Ubah (2019) states that "The study of geometry provides opportunities for learners to visualise concepts that may be related to other areas of mathematics, including trigonometry, patterns and measurement" (p. 2). Effective teaching of Euclidean geometry has proven difficult, as previous studies have reported that many students have difficulty identifying, analysing, and thinking critically about spatial objects and images (Couto & Vale, 2014; Mabotja, 2017). Research indicates that few students tackle these questions and those who do perform poorly (DBE, 2018), even though Euclidean Geometry accounts for 50 ±3 of the marks on Grade 12 Mathematics Paper 2 (DBE, 2011). This being the case, of concern for the current study is that there is dearth of mathematics education research that explored the teaching of the topic at Grade 10 level within the South African context, especially studies focusing on teachers' discourses during teaching. Accordingly, this study sought to explore Grade 10 teachers' teaching of Euclidean geometry, to gain insight into how teachers' explanatory talk during the lessons enabled and/or constrained learners' learning of the topic.

The Grade 12 annual examination reports reveal the difficulties associated with the teaching and learning of Euclidean geometry (Baiduri et al., 2020; Bonnie & Lawes, 2016). Figure 1 depicts the data from a random sample of candidates for the years 2017 to 2019 on their performance in mathematics paper 2, in which Euclidean geometry is included (DBE, 2019). We compiled the poster in Figure 1 from the analysis of different DBE reports, to demonstrate the performance of learners in Euclidean geometry.

Figure 1
Average Percentage Performance per Question for Paper 2 from 2017 to 2019



Q1	Data Handling
Q2	Data Handling
Q3	Analytical Geometry
Q4	Analytical Geometry
Q5	Trigonometry
Q6	Trigonometry
Q7	Trigonometry
Q8	Euclidean Geometry
Q9	Euclidean Geometry
Q10	Euclidean Geometry

Note. The data can be found in DBE (2017, 2018 & 2019).

The depicted performance in Figure 1 demonstrates relative degrees of challenge of each question as experienced by candidates who wrote the Grade 12 mathematics examinations across those years. In relation to Euclidean geometry, the results demonstrate how poorly geometry performance has generally been over the three years. In this regard, the trend, as shown by the above graph, was that students performed well below 50% for questions 9 and 10, with some averages falling well into the 30% range, except for question 8 of 2018, especially when juxtaposed with other questions covering other concepts in paper 2. While the performance reveals the quality of learners' learning and understanding of mathematical concepts, of importance to note is that teachers' pedagogical reasoning and actions during the lessons plays a major role in enabling learners' knowledge retention for assessment. This being the case, the dearth of studies that explored the teaching practices for Euclidean geometry in South Africa has not offered insights into how teachers make the contents available for learners, to ensure high thinking quality and knowledge retention for assessments. Although various reports by the Department of Basic Education have not indicated what it is about the Euclidean geometry lessons learners find difficult, Marange and Tatira's (2023) observation highlights some of the attributes that lead to difficulties in learning the topic. The authors reported that,

> The use of teacher-centered instructional approaches was mostly observed in most secondary schools. It was evident that mathematics teachers encouraged passive learning and made learners feel that they had nothing to contribute because they (teachers) were dominant throughout the lesson. The approach most teachers used led to boredom in class. (p. 2).

This statement suggests that the difficulties learners experience in understanding the Euclidean geometry concepts and subsequent underperformance during examination are because of underteaching of the topic. Thus, the focus for the current study was on how teachers explained concept verbally, what they wrote on the board and notes they handed out as well as how these aided their mathematical explanations related to geometry concepts. The research question that underpinned the current study was: What are Grade 10 teachers' explanatory talk during Euclidean geometry lessons within rural classrooms?

2. Difficulties Associated with Teaching and Learning of Euclidean Geometry

Euclidean geometry instruction is still being neglected, which is a worry (Tutak & Adams, 2015). Wei et al. (2017) contend that teaching Euclidean geometry is challenging because it calls for the use of advanced cognitive abilities, which most teachers lack due to the lack of exposure to geometry concepts as learners and/or under-training at universities to teach the topic effectively. According to Wentzel (2016, p. 49), "Euclidean geometry instruction in South Africa is in a very bad state, and a great number of learners do poorly in Euclidean geometry." This suggests that Euclidean geometry remains a challenging to teach and learn and such challenges are attributed mainly to teachers' teaching practices and the quality of content delivery during teaching (McAndrew et al., 2017; Tutak & Adams, 2015; Utami et al., 2017). For example, Machisi (2021) argues that the difficulties that learners face in understanding geometric proofs are due to the continued overuse of traditional lack of learner-centred approaches during the lessons. Similarly, Sibaya (2020, p. 2) argues that "teachers need to be creative in designing geometric activities that would promote active learning" whereby learners are actively engaged in the process of meaning making during teaching and learning.

While previous studies attribute the challenges associated with Euclidean geometry to teachers' content knowledge and teaching practices, there is scarcity of researcher that explored teachers' teaching of the topic within the South African context, especially with Grade 10 teachers. This is due to the overfocus on learners' errors and misconceptions on the topic, particularly at Grades 11 and 12. As mentioned earlier, this study sought to explore and understand teachers' discourses of Euclidean geometry at Grade 10 level, to address the identified research gap. In addition to the foregoing discussion, findings from Luneta (2015) revealed that most Grade 12 learners underperformed due to conceptual errors they committed when answering geometry questions. This makes it important that mathematics education research focuses on teachers' explanatory talk during teaching, to understand how they make complex concepts available for learners.

Other studies indicated that South African learners, Grade 12 learners in particular are operating way below their envisaged levels, with geometry being one of the challenging topics (Ngirishi & Bansilal, 2019; Siyepu & Mtonjen, 2014). Alex and Mammen (2016) demonstrated that most learners operate at the concrete visualisation level rather than at the expected abstract level in geometry, which requires higher mental functioning for geometrical reasoning. Considering that most math teachers in South Africa were not taught Euclidean geometry as learners as well as in teacher education, it becomes interesting how they facilitate learners' conceptual and procedural development to ensure that they reach higher mental functioning and operate at the abstract level of geometric reasoning (Ugorji & Alfred, 2017).

If students are taught by ineffective teachers, they have very limited chance of comprehending Euclidean geometry, as supported by Wei et al.'s (2017) argument that when teachers possess poor content and pedagogical knowledge of Euclidean geometry, learners are most likely to underperform in the topic. One way of understanding such expertise or lack thereof is through conducting research with teachers, as it is a central premise for the current study that teachers play a critical role in ensuring that learners learn and own knowledge and skills for Euclidean geometry. Researchers such as Ozkan et al. (2018) contend that learners at Grade 12 level experience challenges in Euclidean geometry as a sequel of limited basic skills in mathematics, but little has been done in mathematics education research within the South African context to explore how teachers make those basic skills available for learners during the lessons. The studies reviewed in this section demonstrate the existing challenges in the learning and teaching and teaching of geometry, but we argue that previous studies focused mainly on attributes to underperformance in geometry, overlooking researching with teachers as the key role players in ensuring that learners learn and own mathematical skills and knowledge of Euclidean geometry. Thus, this study addresses this research gap by offering insights into one teacher's explanatory talk during Euclidean geometry teaching at Grade 10 level.

3. Conceptual Framing: Understanding Explanatory Talk

The central premise of the current study is the position that Euclidean geometry teaching is a discursive activity (Adler & Ronda, 2015). According to Adler and Ronda (2015; 2017) and Sfard (2012), mathematics teaching entails processes whereby teachers and learners use different mathematics discourses, in which meaning related to mathematical objects are co-constructed through classroom interactions and language. This means that, mathematical objects derive their existence and associated meanings as teachers and learners interact with mathematical concepts and each other during teaching and learning to construct meanings for such concepts. This perspective posits that mathematics teaching involves engaging in mathematical discourse, whereby mathematical objects are constructed using language and communication (Adler & Venkat, 2014). This resonates with Lynch and Bolyard (2012) definition of mathematical discourse, which they view as the oral and/ or written communication of mathematical concepts or ideas during teaching and learning, the teaching of Euclidean geometry for the current study. One way of elaborating on this perspective is that it is envisaged that mathematics teachers explicitly articulate and create opportunities for discussions of the mathematical concepts to enable learners' mathematical learning (Gresham & Shannon, 2017).

According to Sequeira and Ferreira (2014), frequently, the meanings of words used in everyday speech are ambiguous. Abstract conceptualization is the most basic scientific technique. According to him, conceptualization is the process of defining precisely what terms we use in our research reflect what we mean and don't mean. Additionally, Sequeira and Ferreira (2014) argued that the term "concept" refers to the outcome of "conceptualization," regardless of whether it refers to a single word or a complicated series of related events or ideas. Words or symbols that designate a significant whole are referred to as concepts. Terms that we employ to describe concepts are also concepts. Each idea in the definition must also be understood to properly comprehend the description of the supplied concept.

According to Hiebert and Carpenter (1992), ability to participate in society as good people that is becoming more and more international, one must have a thorough understanding of Euclidean geometry. In this section, I define the term "explanatory talk" in relation to the current research. It is essential for both educators and learners to actively take part of developing one's ability to reason new concepts and body of knowledge. In order to help students, consider about their reasoning and use the best resources to understand newest ideas, teachers must design lessons that make links between students' prior knowledge and their new knowledge (Hiebert & Carpenter, 1992). In the current study, "explanatory talk" is considered to be a discussion that makes an effort to link various concepts in order to comprehend the nature of certain problems and explains the patterns of relationships between them.

Explanatory talk enables for the exploration and understanding of whether a teacher uses appropriate mathematical words and statements in explaining mathematical concepts and/or procedures during teaching. To operationalise the concept of explanatory talk, Adler and Ronda (2015) draw from Bernstein's concept of pedagogic discourse to account for what counts as mathematical in mathematics teaching and learning. That is, the role of explanatory talk during teaching is to name and legitimate the object of learning, the examples teachers select and use as well as how tasks are facilitated during classroom instruction. This tenet focuses on both what teachers write and say during teaching, to provide descriptions of the mathematics teachers make available for learners to learn and own through explanatory talk, as well as making summative judgments on naming and legitimating mathematics concepts and explanations as they accumulate within a lesson and across lessons (Adler & Ronda, 2015). In this study, this tenet of Mathematics Discourse in Instruction [MDI] enables us to evaluate teachers' substantiations of mathematics related to the concept of Euclidean geometry as specialised knowledge, to understand what counted as mathematical knowledge in teachers' elucidations during teaching.

In relation to what counts as mathematically endorsed explanations during teaching, Adler and Venkat (2014) foregrounds the tension in how teachers navigate the complexity of managing the formal and informal ways of describing mathematical concepts. In this study, this tension was observable as teachers were tempted to simplify concepts and words during explanations, considering that the concept of Euclidean geometry is formally introduced for the first time in Grade 10, as a way of giving learners access to mathematical meanings, resulting in the usage of informal ways of describing geometry concepts. Adler and Venkat (2014) contend that often teachers become reluctant "to use formal mathematical language" during teaching as they consider the usage of formal mathematical words and statements as "abstract and the learners are put off by over reliance on formal talk with neglect of connecting mathematical ideas to colloquial meanings"

(p. 132). Accordingly, the notion of explanatory talk enables us to unpack the mathematical discourses teachers inhabited during explanations, to understand how these facilitated learners' learning of Euclidean geometry lessons as they named and legitimated different concepts during the lessons.

4. Methodology

Considering that qualitative research is aimed at providing an explicit interpretation of the broad patterns, order and the structure found among the study participants, observations are the best way to generate depth understanding of the nature of events which participants engage in (Guthrie, 2011). To gain insight into teachers' mathematical discourses during Euclidean geometry lessons, observations "reveal classroom norms about teachers' authority, implicit rules about pupil participation, and the structure of classroom work and tasks" (Guthrie, 2011, p. 87). In this study, we used unstructured classroom observations that were non-participatory in nature. This allowed us to understand teachers' discourses from their pedagogical patterns during the lessons. Instead of imposing predetermined assumptions about teachers' discourses of Euclidean geometry, the unstructured nature of the observations allowed the observations of teachers' teaching naturalistically (Cohen et al., 2013). In other words, using unstructured classroom observation technique enabled us to "postpone definitions and structures until a pattern emerged", out of the discourses that the four teachers1 inhibited during teaching and learning (Bell, 2005, p. 185). We adopted Mbhiza's (2021, p. 82) approach to allow "the trends and patterns reliably emerged out of how teachers acted, what they said during teaching and how they interacted with learners, mathematical contents and other physical artefacts in the classrooms" in observing lessons on Euclidean geometry. As Mbhiza (2021) suggests, we did not engage in any active participation during teaching and learning such as interjecting. We focused on what we observed during the lessons and in turn made interpretations and summative judgements about teachers' explanatory talk during Euclidean geometry lessons. All classrooms' observations were video recorded, to ensure that we captured sufficient information to understand Grade 10 teachers' explanatory talk during Euclidean geometry lessons.

The researchers' non-participatory presence during the lessons minimised disruptions during teaching and learning of Euclidean geometry. This being the case, it should be noted that our presence in the classrooms during the lessons has somewhat impacted on the naturalistic way teachers and their learners interact when there is no 'stranger' in the classroom space, especially the influence of having a video camera during the observations. Cohen (2011) referred to this as particpants' reactivity, in which research participants alter their actions based on the presence of a 'stranger' in their setting, sometimes to impress the observer. Of importance to note is that, with time, the level of what we considered reactivity lessened, which was evedenced by the teachers and learners no longer focusing or giving gaze to the camera during classroom interactions.

We also used Video-Stimulated Recall Interviews [VSRI] to allow teachers to engage in reflective conversations and we provided them with feedback on their explanatory talk and how they facilitated and/or constrained learners' learning of Euclidean geometry. In this paper, we focus primarily on the data generated from classroom observations.

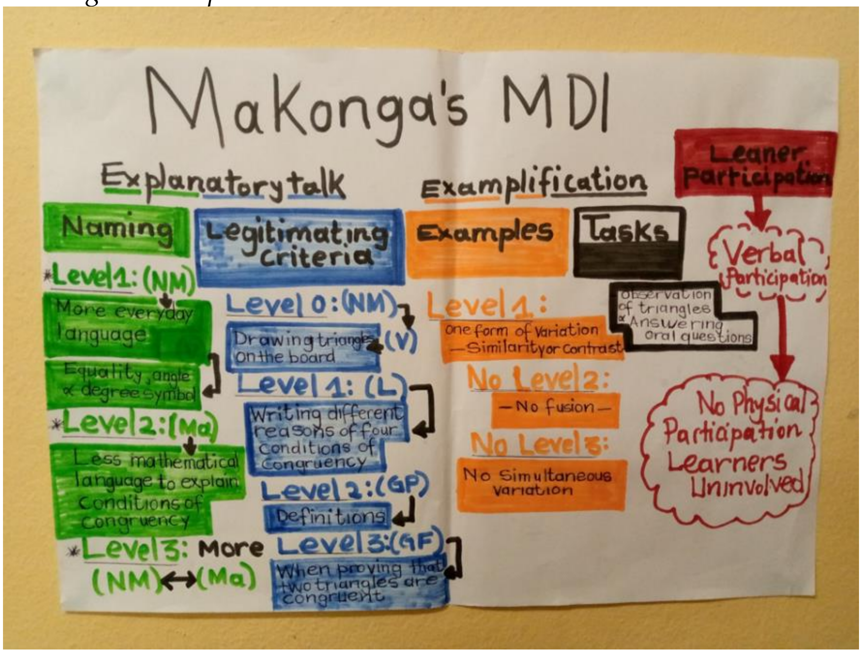
Within non-probability sampling, we employed purposive sampling to select four the teachers from four different school sites for this study. According to Cohen et al. (2013), purposive sampling consists of handpicking of participants according to characteristics they have and required by the study, which includes their envisaged knowledge and abilities to answer the research questions. Thus, purposive sampling was employed in this study because participants were not only expected to possess a characteristic of a mathematics teacher but characteristic of being a Grade 10 mathematics teacher who has experiences of teaching Euclidean Geometry. In

[1] In the study we researched with four Grade 10 mathematics teachers. For the current paper, we selected only one teacher's lesson to offer in-depth analysis and interpretations of their explanatory talk during teaching.

this paper, we focus on one teacher's teaching as this enables us to zoom in-depth into their explanatory talk during teaching.

Maree (2007, p. 99) states that "qualitative data analysis is usually based on an interpretive philosophy that is aimed at examining meaningful and symbolic content of qualitative data." He indicates that, to understand how participants make meaning of a phenomenon, it is important to analyse their perceptions, attitudes, knowledge, feelings, and experience. According to Cohen et al. (2013, p. 462), qualitative data analysis is a process of "making sense of the data in terms of the participants definition of a situation, noting patterns, themes, categories and regularities." This suggests that qualitative data analysis is concern with reliable interpretation of the information provided by the participants. Within qualitative data analysis, this study used content analysis for all the data sources both individually and relationally. The rationale for choosing content analysis is it enabled us to interpret data is through coding and categorising it to see similarities and differences, to interrogate teachers' discourses to formulate summative judgements about their teaching of Euclidean geometry (Adler & Ronda, 2015; McMillan & Schumacher, 2010). All the recorded lessons were summarised using the components of MDI discussed earlier. Figure 2 is poster chats showing the process of generating the summative judgements for each teacher.

Figure 2
Makonga's MDI poster charts



As we watched different teachers' lessons, we engaged in discussions relating to the nature of the teachers' explanatory talk during teaching and we chunked each lesson into episodes that are identified by different objects of learning. Chunking produced numerous episodes and we then examined each episode based on the MDI summative judgements tool, focusing on their explanatory talk during the lessons. In this paper we focus on only one teacher's selected lesson, do demonstrate how their explanatory talk during teaching limited learners' learning of geometric concepts and principles.

5. Results

In this study, we sought to explore and understand Grade 10 teachers' mathematical discourses during Euclidean geometry lessons, and this paper focuses on only one element of mathematics discourse in instruction, teacher's explanatory talk. The four conditions of congruency that we present below were not identified as a result of the analysis of the lessons but represent how the teacher organised his lesson during teaching. In this section, we present and analyse Makonga's nature of explanatory talk during teaching, to understand how they made mathematics contents relating to Euclidean geometry available to the learners. Of importance to note is that the presentation of the conditions of congruency of triangles represent segments from one whole lesson, there was no time lapse between the segments.

5.1. Conditions of Congruency of Triangles

To start the lesson in focus, Makonga introduced and described the conditions of congruency of triangles. It took him taking almost 5 minutes checking in his textbook the page number on which the conditions of congruency of triangles are displayed while his learners were waiting. This demonstrates that Makonga was not ready for this lesson and the only thing he could do was to be attached to his textbook and to be glued to the whiteboard (See Figure 3).

Figure 3
Makonga standing with his textbook before his learners

He started the lesson by the following statement:

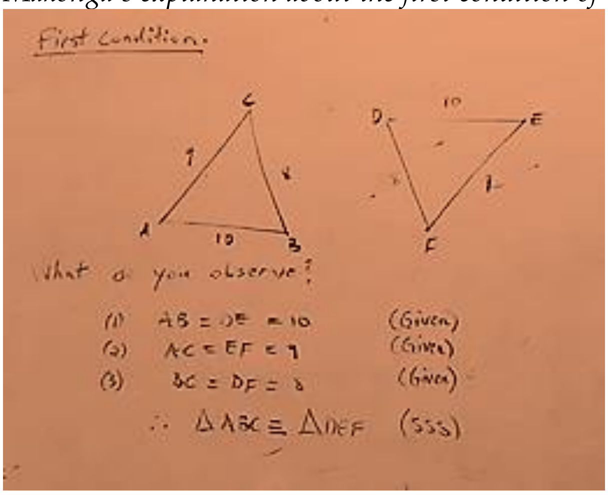
> When we talk about congruency of triangles, you need to know that there are four conditions that need to be satisfied for triangles to be congruent. We are going to start with the first condition.

Makonga started the lesson without doing a recap of the background knowledge. He explained and described the conditions of congruency triangles as illustrated below.

5.1.1. First condition

He started by drawing two triangles on the whiteboard and then engaged learners with some oral question as in Figure 4.

Figure 4
Makonga's explanation about the first condition of congruency



After drawing these two triangles on the whiteboard, Makonga proceeded with the following discussion:

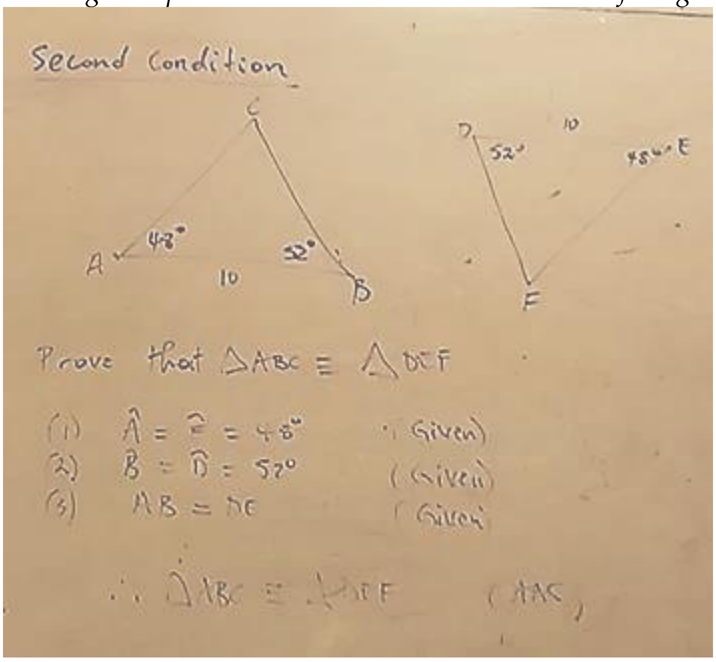
1 Makonga: *What do you observe in this diagram?*
2 Learner 1: *AB = DE = 10*
3 Learner 2: *AC = EF = 9*
4 Learner 3: *BC = DF = 8*
5 Makonga: *What conclusion can you give?*
6 Learner 4: $\Delta ABC \equiv \Delta DEF$
7 Makonga: *Why?*
8 Learner 4: *Because the corresponding sides of triangles are equal.*
9 Makonga: *How do you write this condition?*
10 Learner 5: *SSS*

The use of individual responses was interesting to watch because it illustrates that the questions that the teacher asked in those instances signalled a level of understanding, as learners had to provide answers relating to what they have observed in the diagram. This exchange further illustrates the observable action of drawing two triangles with their measurements has help learners to see that the two triangles were equivalent. However, we have observed in lines 2, 3 and 4 that Makonga did not ask learners to provide reasons to their statement even though the measurements were given on the diagram, he should have encouraged learners to provide the reason "GIVEN" so that they get use to the way of answering questions in Euclidean geometry.

5.1.2. Second condition

He used the same approach of starting by drawing two triangles on the whiteboard and then engaged learners with some oral question. Figure 5 represents the second condition of the congruency.

Figure 5
Makonga's explanation about the second condition of congruency



After drawing these two triangles on the whiteboard, Makonga proceeded with the same routine of engaging learners in a discussion as illustrated below.

11 Makonga: *What do you see?*
12 Learner 1: *The two triangles are congruent.*
13 Makonga: *How is that possible?*
14 Learner 1: *Because the look the same in terms of shape.*

15 Makonga: *No. Who can give us reasons that are more concrete?*
16 Learner 2: $\hat{A} = \hat{D} = 48°$
17 Learner 3: $\hat{B} = \hat{E} = 52°$
18 Learner 4: $AB = DE$
19 Makonga: *Correct. Now we can be able to say that* $\Delta ABC \equiv \Delta DEF$.
What reason can we give to support this statement?
20 Learner 5: AAS
21 Learner 6: *What about the following condition: AAA?*
22 Makonga: *It is not a condition for congruency, but this is a condition for similarity.*

The exchange above demonstrated how Makonga provided sufficient learning opportunities, which are somewhat aligned with the learning goals, and engaged majority of the learners to participate, to cooperate, and to collaborate in continued learning. However, we noticed that Makonga did not take time to comment on learners' responses. For example, in line 12, Learner 1 provided a correct answer by saying "*the two triangles are congruent*" but Makonga could not agree with the learner since the answer provided was not the one that he was expecting; in line 14, Learner 1 gave the reason why the two triangles were congruent by saying *"because they look the same in shape"*. That is, in lines 12, Learner 1 gave a correct answer that required mathematical justification.

To continue with learner interaction, Makonga responded positively by tailoring subsequent question, requiring the learner to explain their answer, but then dismissed their response in line 14 and asked for a more concrete explanation. Instead of dismissing the learner's response, the teacher could have scaffolded their learning by asking guiding questions as the More Knowledgeable Other [MKO] (Vygotsky, 1987), such as "What makes you think that they look the same?" This could have sustained the learner's thinking about mathematical concepts in focus. This being the case, other learners in lines 16-18 provided the expected answers. This event demonstrates the need for teachers to maintain substantive engagements with the learners even in cases where learners give incorrect or answers that teachers did not expect. This is one-way teachers can extend and/or disrupt learners' current learning and mediate their thinking for future learning (Mbhiza, 2021).

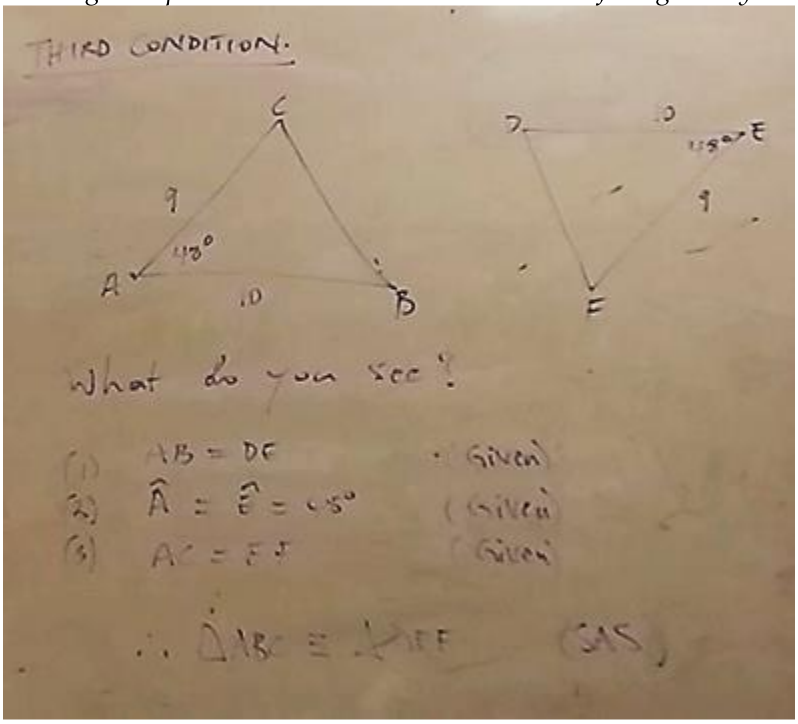
In my observation, the answer provided by the learner was not wrong in the fact that the learner used everyday language (*colloquial naming*) instead of mathematical language to describe his observations about the relationship between the two triangles. According to Adler and Ronda (2015), teachers are tasked with ensuring that learners learn and own skills to navigate between everyday and formal mathematical ways of thinking and speaking about mathematical objects. It is the role of the teacher to teach learners mathematical ways of answering questions, ensuring that pedagogical links are made to support connections between everyday ways of explanations and mathematical ways of explaining relationships in mathematics learning (Scott et al., 2011). Furthermore, Makonga did not comment on the responses that learners provided in lines 16, 17 and 18 even though the answers were mathematically correct, but they did not give any reasons to support their answers.

Furthermore, to support knowledge building, Makonga could have provide explanatory talk relating to the second condition of AAS or SAA in words and offered a counterexample for learners to make observations and move towards generality (Adler & Venkat, 2014). According to this condition, two triangles are congruent if two angles and one side of a triangle are equal to two angles and a ***corresponding side*** of the other triangle. Providing a counterexample can help in mitigating a common misconception where learners mistakenly consider two triangles congruent because of the two equal angles and one side. The naming in line 22 was also interesting, that Makonga juxtaposed between conditions for congruency and conditions for similarity but did not demonstrate the differences further for the learners through explanatory talk or using visual aids to show their differences. This could have enabled learners' in-depth understanding of the differences between the two and build relational links between the two concepts (Skemp, 1976).

5.1.3. Third condition

This condition is introduced similarly to the previous two conditions. In line 33, Learner 6 asked if the order of sides matters. The teacher emphasized that the order of sides does not matter. He drew once again two triangles on the whiteboard as depicted in Figure 6 and then engaged learners with some oral question as presented in lines 23-34.

Figure 6
Makonga's explanation about the third condition of congruency



23 Makonga: *What is your observation on this diagram?*
24 Learner 1: *AB* = *DE*
25 Makonga: *Correct. Anyone else?*
26 Learner 2: $\hat{A} = \hat{E} = 48°$
27 Makonga: *Good job. One more thing.*
28 Learner 3: *AC* = *EF*
29 Makonga: *Good. That is enough to conclude that* $\Delta ABC \equiv \Delta DEF$. *What reason can we give to support this statement?*
32 Learner 5: *SAS*
33 Learner 6: *Sorry Sir, does the order of sides counts in this condition?*
34 Makonga: *Yes, it does. You need to have two sides and one angles between those two sides. Nothing else is acceptable.*

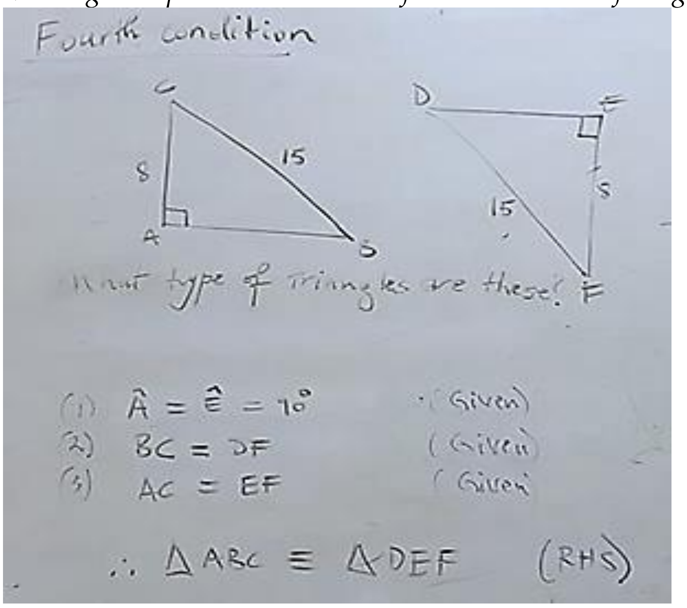
In the conversation above, Makonga demonstrated accurate knowledge of key concepts both in terms of the knowledge of the content and knowledge of his learners and he responded to learners' questions or comments, creating a learning environment that allows dialogic interactions between the teacher and learners (Adler & Venkat, 2014). The only negative aspect we have observed is that Makonga did not once again encourage learners to provide a reason to the statements in lines 24, 26 and 28. In addition, he did not adapt and modify learning opportunities to create a supportive learning environment for learners to recognize each other's learning strengths, and value the contribution of others due to the structure of the classroom and the large number of learners in his class.

5.1.4. Fourth condition

Makonga drew once again two triangles on the whiteboard (see Figure 7). This time the two triangles where completely different from the previous cases and then engaged learners with some question-and-answer process (lines 35-52). It could be said that Makonga's discourse of teaching Euclidean geometry is the naming of concepts through question-and-answer strategy, which is common among all the participating teachers in this study. What sets Makonga's teaching from the other teachers' is that he made some movements towards engaging with leamers' questions during teaching and offered some explanations relating to the concepts he introduced. While in the teaching of the third condition learners only used one-word responses to answer the teacher's questions, in teaching the fourth condition, Makonga encouraged interpretive elaborations through asking questions such as 'what else?'

Figure 7

Makonga's explanation about the fourth condition of congruency



35 Makonga: *Before describing the last condition. What types of triangles are displayed in the diagram?*
36 Learner 1: *Right-angled triangles*
37 Makonga: *Correct. What are the characteristics of such triangles?*
38 Learner 2: *They contain a right angle.*
39 Makonga: *Good job. What else?*
40 Learner 3: *It has two adjacent sides to the right angle and one longue side called hypotenuse.*
41 Makonga: *Do you still remember the last condition?*
42 Learner 5: *RHS*
43 Makonga: *Correct. Let us now prove that these triangles are congruence. What do you observe from the diagram?*
44 Makonga: *Yes, it does. You need to have two sides and one angle between those two sides. Nothing else is acceptable.*
45 Learner 6: $\hat{A} = \hat{E}$
46 Makonga: *What is the reason?*
47 Learner 6: *They are both equal to* 90°
48 Makonga: *What else do we have?*
49 Learner 3: *BC=DF*
50 Learner 4: $AC = EF$
51 Makonga: *Correct. Now we can be able to say that* $\Delta ABC \equiv \Delta DEF$*, with RHS as the reason.*

The last condition of congruency was well explained by Makonga with more interaction between him and the learners. We have observed that Makonga used only oral questions to engage learners during the lesson and did not give neither opportunities to learners to go to the board and demonstrate their ability and skills nor a chance for them to exchange ideas between themselves. It is important to note that even in this event, Makonga did not request learners to provide reasons for their mathematical statements, "so the criteria for how and why this was legitimate" was not created (Adler & Ronda, 2017, p. 14).

6. Discussion

This theme focuses on Makonga's lack of explanatory talk during Euclidean geometry lessons, in which what was considered legitimate was produced in learner responses to teacher's questions, with no accompanying elucidations from him. That is, while the Makonga allowed the learners to provide naming for various Euclidean geometry related concepts, specialised meanings for various concepts remained implicit due to lack of his explanatory talk. The overuse of question-and-answer discourse during the lesson resulted in lack of in-depth legitimation and naming of mathematical concepts related to Euclidean geometry. According to Sfard (2019, p. 1), "it is a common lore that teachers bear the main responsibility for what the students learn or fail to learn", suggesting their influence regarding learners' understanding or lack thereof for knowledge. Makonga presented the information using visual cues and used the question-and-answer strategy to get learners to identify and name Euclidean geometry concepts and legitimate specific narratives. Of importance to note is that teachers' explanations play a major role in bringing the object of learning into focus.

One thing that research that focus on the notion of instructional explanations in mathematics is silent about is that explanatory talk is not complete after the teacher just presented the information about the mathematical objects. Equally, just asking learners to recall and verbalise what they know about mathematical objects throughout the lessons without the teacher building on their responses for further elaboration limited the effectiveness of teachers' mathematical discourse in this study. This raised a question for me throughout this study, what does explanatory talk mean? What dominated Makonga's teaching in the current study were the lack of sustained explanations for him to present and legitimise Euclidean geometry concepts and processes. Thus, the lack of teacher explicit and engaged talk, beyond the questions and confirmations of learners' answers limited opportunities for foregrounding "what is to be known or done, and how" (Adler & Ronda, 2017, p. 6).

Although it could be argued that the discourse of question-and-answer that was predominant in the Makonga's lessons resonate with the notion of teachers becoming facilitators of mathematical contents rather than being at the centre of teaching and learning, having no explanations, elucidations on how learners should work with specific concepts and associated rules can limit learners' epistemological access to geometry concepts (Charalambous et al., 2011). This is concerning considering that the more abstract geometry concepts are introduced at Grade 10. Makonga overlooked the language level engagement in his exchanges with learners, that as much as learners are the ones who should learn and own Euclidean geometry concepts and skills, he ought to make naming and legitimating mathematical statements to guide learners about the nature of mathematics concepts and their relatedness (Wittwer & Renkl, 2008).

Makonga's pedagogical actions related to explanatory talk are contrary to Tachie's (2020) findings that, some teachers spoke and wrote throughout the teaching and learning process without allowing learners opportunities to internalise the contents presented and discussing their understanding thereof. Makonga was the one who engaged in peripheral teaching, in which he posed questions and prompted learner responses until the expected answers were verbalised. Makonga dominated the writing on the board like in Tachie's study, but he did not present explanations to guide learners on how to work with different geometry concepts and principles that he introduced. Accordingly, we argue that questioning learners about mathematical objects and confirming or disconfirming their answers cannot be the endpoint of teaching. It should be noted that Makonga did not use 'responsive explanations', to unpack concepts that presented some difficulties for learners, as he focused primarily on using prompts until correct answers to questions were verbalised by learners.

7. Conclusion

This paper has demonstrated how Makonga allowed his learners to provide naming for different concepts they introduced in the lesson, but he did not offer explanatory talks to bring the specialised meanings to the fore and legitimate concepts and meanings for the learners. He predominately used questions-and-answer strategies to engage with different topics. While this way of teaching can be a good strategy to get learners to become active co-constructors of Euclidean geometry concepts, we argue that teachers' explanations remain vital to ensure concepts clarification and legitimation for learners. Over-using the question-and-answer discourse during teaching resulted in lack of in-depth legitimation and naming of mathematical concepts related to Euclidean geometry which is concerning considering that this topic has numerous concepts that require clear operationalisation by the teacher for learners to learn and internalise such concepts. Overall, lack of teacher explanations in Makonga's teaching limited opportunities for epistemological access to Euclidean geometry concepts and associated principles.

8. Recommendations

We have realised that teaching Euclidean geometry effectively requires effective mathematics explanatory talk. We recommend that teachers should begin a lesson with fundamental concepts like points, lines, and angles to ensure that learners should have a solid foundational knowledge before moving on to more complex topics. This means that, teachers should make links between concepts covered in earlier grades to help learners with knowledge building and creation of the mathematics story related to the topic.

Teachers should also consider introducing Euclidean geometric concepts with real-world examples to make abstract ideas more tangible and relatable for learners. They should also limit the overuse of questions-and-answer discourse when teaching Euclidean geometry, especially at the beginning of the concept at Grade 10. Instead, teachers should use a variation of examples and offer explanatory talk to help learners develop the language for Euclidean geometry. Equally, learners should be encouraged to ask questions about Euclidean geometric concepts, and they should be able to model effective questioning techniques to stimulate curiosity and critical thinking. Teachers should pose challenging problems that require critical thinking and problem-solving skills and prompt learners to engage in sustained conversations about Euclidean geometry principles and theorems. This means that teachers should facilitate open-ended classroom discussions about Euclidean geometric concepts, encourage learners to justify their reasoning and engage in respectful debates to help them towards generality of the concepts and related processes.

Opportunities for future research include, explore how the different components of MDI influence each other in bringing the object of learning into focus; and explore how teachers professionally notice their learners' mathematical thinking and communication of Euclidean geometry concepts and principles during teaching.

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