Journal of Pedagogical Sociology and Psychology
Volume 6, Issue 3, 2024
https://doi.org/10.33902/jpsp.202430538

Research Article

Mathematical literacy of primary school students in the context of mathematical modeling

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Received 25 August 2024; Revised 30 October 2024; Accepted 19 November 2024

This study aimed to investigate the mathematical literacy skills of fourth-grade primary school students in the process of mathematical modeling and their views on the modeling process. Mathematical modeling activities were carried out with students for four weeks in this context. The study group consisted of 15 fourth-grade students. Students' solution and report papers, video recordings, and students' diaries were used to collect the data, and the data underwent content analysis. The results showed that fourth-grade primary school students have a moderate level of mathematics literacy skills. Within the realm of skills in mathematical literacy, students had the highest scores in communication, representation, and problem-solving strategies creation skills. Conversely, their scores in symbolic skills were the lowest. It was determined that as students engaged in modeling activities, they had an increasing sense of comfort and proficiency, resulting in more efficient problem-solving. Students predominantly highlighted the modeling process in terms of the promotion of values such as teamwork, collaboration, and solidarity. It is possible to assert that mathematical modeling activities facilitate students' cognitive processes as well as provide them with emotional and social support.

Keywords: Mathematical literacy, mathematical modeling, primary school students

1. Introduction

Mathematics is a field of study that possesses its own unique system of communication. Right from the outset, children should commence utilizing this language accurately and effectively. The objective is to cultivate children into mathematically proficient persons as soon as they are introduced to mathematics in a program. When the literature is examined, it becomes clear that the early childhood years are the stages during which development and learning take place at a quick pace (Charlesworth & Lind, 1990; Cole & Cole, 2000). Given the importance of these periods for acquiring knowledge, it has been noted that the fundamental principles of mathematical abilities are developed during early childhood, encompassing both preschool and primary school (Brewer, 2001; Clement & Sarama, 2007). Suharta and Suarjana (2018) argue that mathematical literacy should serve as the fundamental basis for mathematics education. Primary school students will acquire fundamental mathematical principles that enable them to use mathematics in various societal contexts (Akman, 2002). This circumstance is likewise a commendable endeavor aimed at enhancing mathematics literacy (MoNE, 2018). Mathematical literacy is a key factor in assessing the effectiveness of primary school education (Isa & Murni, 2018; Saleh, Prahmana). However, pupils' capacity to utilize their mathematical literacy in progressing phases is determined by their experiences in this respect. Therefore, initiating the cultivation of mathematical literacy from the primary school level is crucial (Csíkos & Verschaffel, 2011).

2. Literature Review

2.1. Mathematical Literacy

Mathematical literacy is becoming increasingly critical in the 21st century, and it is deemed essential for students to be able to keep up with the modern era (Fauzi & Chano, 2022; Geiger, Goos & Forgasz, 2015; Rizki & Priatna, 2019). The mathematics teaching systems in several countries, such as Turkey, the Netherlands, Belgium, Canada, Finland, Norway, Ireland, and Japan, emphasize the importance of mathematical literacy and incorporate specific skills to enhance this literacy (TIMSS, 2015). Upon examining the programs, it is evident that students are required to utilize their predictive abilities proficiently in mathematical literacy. Nevertheless, the programs strive to enable students to attain a proficiency level where they can comprehend mathematical principles and effectively utilize these principles in their daily lives (MoNE, 2018; MECSST, 2008; NDET, 2013). Similarly, PISA expects pupils to achieve a minimum proficiency level of at least the second level in mathematical literacy (OECD, 2022). Students at this stage are developing the capacity to apply mathematical concepts in basic real-world scenarios (OECD, 2023). At this point, the inquiry into the definition of mathematical literacy has become significant.

Multiple definitions of mathematical literacy can be found in the literature. The International Life Skills Survey (ILLS) defines mathematical literacy as the comprehensive set of competencies, including skills, beliefs, knowledge, problem-solving, and communication, that individuals require to effectively navigate mathematical issues in their daily lives (MCATA, 2000). PISA defines mathematical literacy as the aptitude to apply mathematical concepts to address personal life requirements and the capability to actively participate in mathematical activities. When discussing the significance of mathematics in life, it is important for an individual to rely on well-founded reasoning (OECD, 2003). Again, the OECD (2013) defines mathematical literacy as the capacity to construct, employ, and comprehend mathematics, it emphasizes that this competence encompasses the application of mathematical reasoning, concepts, methods, facts, and tools to describe, elucidate, and forecast incidents.

Mathematical literacy includes a range of mathematical abilities, such as logical thinking, recognizing the societal implications and advantages of mathematics, and comprehending the essence and evolution of mathematics. It also involves the practical use of mathematical skills. PISA aims to assess the extent of mathematical literacy by giving real-life problems to students, allowing them to use their mathematical skills (Altun, et al., 2018; Wilkins, 2000). To discuss the skill of mathematical literacy, it is expected that an individual has specific mathematical competencies. These competencies are taken into account while evaluating mathematical literacy. Mathematical competencies refer to the capacity to use mathematical thinking to solve real-life problems, including logical and spatial reasoning. This includes understanding, evaluating, applying, and utilizing mathematics in different situations and contexts where it is relevant. It also involves the ability to use formulas, models, constructs, graphics, and tables. Mathematical competence is defined as a distinct primary component of mathematical skill (MoNE, 2018; Niss, 2003).

When examining the various forms of mathematical competencies, it becomes evident from numerous sources that these forms encompass mathematical thinking, modeling, devising problem-solving strategies, reasoning, representing, communicating, employing symbolic and technical language and operations, and utilizing mathematical tools (Albaladejo, García & Codina, 2015; Liakos, Rogovchenko & Rogovchenko, 2018; NCTM, 2000; OECD, 2016). Turner, Blum, and Niss (2014) define mathematical literacy within the scope of various competency areas which are representation skills, communication skills, reasoning skills, problem solving strategies skills, modeling skills, and symbol skills. These competency areas, proposed by Turner et al. (2014), served as this study's framework for examining mathematical literacy skills. Providing students with real-life problems to utilize the mathematical competencies mentioned above plays a facilitative role in the development and assessment of mathematical literacy (Sumirattana, Makanong & Thipkong, 2017). Mathematical modeling is one of the implementations that can be used from this point forward.

2.2. Mathematical Modeling

During the process of mathematical modeling, students engage in activities where they model and solve problem situations that they might experience in real life. Subsequently, they apply the solutions they have developed to real-world circumstances. In mathematical modeling, real-world issues are transformed into mathematical problems by establishing specific assumptions, resulting in the development of a mathematical model of the problem. Subsequently, the solution to this mathematical problem is assessed for its potential application in real-world scenarios. Thus, modeling activities would offer numerous opportunities for individuals to improve their analytical abilities (Berry & Houston, 1995; Lesh & Doer, 2003). Mathematical modeling is an approach that enables individuals to identify the relationships that are inherent in the problems they might encounter in life and to articulate these relationships in mathematical terms. It helps individuals reach conclusions through the process of classification and generalization (Fox, 2006). Lingefjard and Holmquist (2005) describe mathematical modeling activities as a highly effective approach that enables students to recognize and understand the various aspects of mathematics in real life, rather than solely as a means for students to learn mathematics. These characteristics of mathematical modeling establish appropriate conditions for the assessment of mathematical literacy.

2.3. Mathematical Modeling and Mathematical Literacy

When considering that mathematical literacy involves students' ability to apply mathematics to real-life problems and comprehend the significance of mathematics, it is evident that it also includes mathematical modeling. Similarly, NCTM (2000) states that the modeling process is demonstrated by students' awareness of the relationships between mathematical ideas outside the world of mathematics, their understanding of these ideas, and their use of mathematical skills. According to Mumcu (2016), individuals who are capable of applying mathematics to real-world scenarios demonstrate a high level of mathematical literacy and possess the ability to model mathematical concepts. Providing students with examples in mathematical modeling tasks that allow them to make multiple links facilitates the development of enduring associations with real-life situations. This situation is a factor that enhances mathematical literacy (Swan, Turner &Yoon, 2006). Nevertheless, an examination of literature related to mathematical modeling (Canbazoğlu & Tarım, 2023; English, 2006, 2012; Kaygısız, Şenel & Kaygısız, 2021; Şahin, 2014; Şahin & Eraslan, 2016, 2017, 2018; Ulu, 2017; Wei, Zhang & Guo, 2022) and mathematical literacy (Çilingir & Artut, 2017; Diputra, Suarjana & Japa, 2019; Erişen, 2022; Firdaus & Herman, 2017; Kurniawati, Gunawan & Marlina, 2020; Şenol, 2022) reveals an insufficient number of studies conducted at the primary school level. However, it is crucial for children to have the ability to connect mathematics with real-life situations starting at a young age. This is essential for them to become mathematically proficient persons who can effectively utilize their knowledge in solving everyday problems (MoNE, 2018; OECD, 2016). In this situation, it would be advantageous to implement mathematical modeling activities at the primary level (English, 2006; Şahin & Eraslan, 2018). Similarly, while analyzing the PISA data, it becomes apparent that Turkey's proficiency in mathematical literacy falls behind many countries (MoNE, 2023; OECD, 2023). Therefore, it is imperative to prioritize modeling studies in primary school to enhance mathematical literacy skills (English, 2012). With the use of mathematical modeling, an environment in which many skills such as investigation, inquiry, collaboration, communication, and reasoning skills are activated (Helding et al., 2010). Students develop different problem-solving strategies in the modeling process, adopt different perspectives, and acquire the skills of using mathematical language while working with the modeling activities presented to them (Biembengut & Hein, 2010; Lesh, 1985). Among the fundamental skills deemed as a component of mathematical literacy is mathematical modeling, which also enables the implementation of other fundamental skills deemed part of mathematical literacy, such as reasoning and communication skills, through the modeling process (Turner et al. 2014). This bidirectional structure points out the interrelated nature of mathematical modeling and mathematical literacy.

This work has been guided by the scarcity of research in the literature on mathematical literacy and mathematical modeling in primary schools, as well as the recognition of the significance of early mathematical literacy and mathematical modeling in this environment. Although mathematical modeling is one of the basic skills addressed within the scope of mathematical literacy (Turner et al. 2014), the limited number of studies on primary school students directly linking mathematical modeling with mathematical literacy in the literature has led to focus on this relationship and examine mathematical literacy within the context of the mathematical modeling process to overcome this deficiency. Thus, it is thought that this research will provide insight into whether it is possible to implement mathematical modeling activities and to examine mathematical literacy within the context of the mathematical modeling process, especially in the early grades. Accordingly, this research aims to investigate the mathematical literacy of fourth-grade primary school students in the process of mathematical modeling and their views on the modeling process. Consequently, the aim is to ascertain the mathematical literacy level of primary school students in response to real-world problems and the skills in which they will thrive. The results of this research are expected to help in the identification of the skills that need support to enhance the mathematical literacy of primary school students. Thus, they will serve as a guide for future research in this area and make a valuable contribution to the field of mathematical literacy. In addition, it is believed that it can shed light on the field in terms of how primary school students will experience modeling activities, and as stated above, whether the opportunity to observe mathematical literacy in modeling activities will be captured. In this context, the answers to the following research questions have been sought:

RQ 1) What is the level of mathematical literacy of fourth-grade primary school students in the mathematical modeling process?

RQ 2) What are the views of fourth-grade primary school students on mathematical modeling?

3. Method

As the purpose of this study was to conduct a comprehensive examination of the solutions that students presented in the context of mathematical modeling activities and their views on the process, a case study design was implemented in this study (Creswell & Creswell, 2013).

3.1. Participants

The study was carried out with fourth-grade primary school students enrolled in a private school in Türkiye. The study group consisted of 15 students (5 girls and 10 boys) who willingly participated in this research. Students possess diverse academic levels. Of these students, 5 had high academic achievement, 7 had medium academic achievement and 3 had low academic achievement. The students are the children of families with high socioeconomic status.

The study is designed by using the convenience sampling approach. The researcher has selected the private school in which she works for. Private schools and public schools have distinct differences in terms of their physical, social and academic characteristics. The study was conducted at a school that works on a campus system. Students attend courses with their primary classroom teachers as well as additional subject teachers, predominantly in the English language. At the school where the study is done, courses are extended by an additional two hours in comparison to public schools. While students are instructed in fundamental topics such as mathematics, Turkish, social studies, and science by their classroom teacher, subjects like English, physical education, music, art, drama, chess, swimming, religious education, and German are taught by specialized subject teachers.

3.2. Data Collection

The research utilized various data collection instruments, including students' solution and report papers, video recordings, and students' diaries. Students' solution and report papers include the collective solutions generated by the students throughout mathematical modeling activities, along with the accompanying reports detailing their thought processes throughout the activities. After each modeling session, these documents were gathered from all modeling groups. Within this context, a total of 20 solution and report papers have been gathered upon the completion of a four-week mathematical modeling process, originating from five distinct modeling groups. During the mathematical modeling processes, the stages of producing solutions to the mathematical modeling activities of some groups and all solutions presented by the modeling groups in the classroom were recorded by video. A total sum of 400 minutes of video recordings has been acquired, with an average duration of around 100 minutes every week. Furthermore, following the completion of each modeling activity, we gathered diaries including the students' own reflections on the modeling activity and the process involved.

3.3. Procedure

During the implementation phase of mathematical modeling activities, the students were sorted into five groups, each consisting of three individuals. The preferences of the classroom teacher were considered in this process. Two of the groups have been chosen as the focus groups. To provide a more comprehensive and multi-faceted representation of the modeling process, one focus group has remained constant while the other focus group has been altered weekly. The process has been scheduled for four weeks, and the researcher has predetermined the modeling activities to be conducted each week, together with the identification of potential solutions. While choosing modeling activities, it was kept in mind that problems should be contextually similar to those that primary school students would encounter in their everyday lives. Furthermore, it was considered that the modeling activities chosen should be interesting and understandable for fourth-grade pupils, as well as appropriate for their age level. The modeling activities used throughout the process are listed in Table 1.

Table 1
Mathematical Modelling Activities

The Implementation Process	*Mathematical Modeling Activities*
1st Week	The Holiday Problem (Doerr and English, 2003)
2nd Week	The Family Dinner Problem (English, 2007)
3rd Week	The Restaurant Problem (Doerr and English, 2003)
4th Week	The Airport Problem (Chamberlin and Coxbill, 2012)

The adapted version of the Holiday Problem was derived from Şahin's (2014) study. The adaption of other activities employed in the research into Turkish was carried out by the researcher. Following that, the final version of the problems was developed by seeking input from five classroom teachers, including a primary school mathematics teacher, a Turkish language teacher, an English teacher, and an academic expert in the field of classroom education. Their expertise was utilized to ensure the problems' clarity and appropriateness for the students' proficiency level. Each week, a mathematical modeling activity was carried out with all groups. The researcher began by reading the problem situation, and then the groups independently started the modeling process by reading it themselves. The constant and changing focus groups were separately video recorded while engaging in the mathematical modeling activities. Upon the completion of the modeling process and the conclusion of the reporting phase, each group delivered their solutions to the class, which were also documented through video recording. Following the presentations, the students were instructed to keep a diary to assess the events of the day. Students have documented their emotions and reflections on the process in diaries, and this practice has been repeated for four weeks.

3.4. Data Analysis

The data was transcribed and underwent content analysis. The framework to reveal mathematical literacy skills developed by Turner, Blum, and Niss (2014) was utilized to analyze the solution papers and reports produced by the groups. In this framework, six categories of competence exist to illustrate mathematical literacy skills. These categories encompass cognitive abilities such as logical thinking, effective communication, creating models, solving problems, representing information, and utilizing symbolic and technical language. Each area is scored at four different levels. Therefore, the analyses of mathematical literacy in this study were based on these competency areas. Each competency area is scored at four levels. The scoring begins at zero and ascends to three. Level 0 serves as an indicator that the competency in a qualification area has either been observed at a minimum level or not at all. Level 1 requires the observation of simple-level behaviors for a specified competency area. Transitions occur from fundamental actions to sophisticated behaviors about the competency domain at Level 2. The highest competency level in a competency area is coded as three. At Level 3, high-level behaviors are observed and complex relationships related to the competence area are established. The researcher conducted a follow-up analysis two weeks following the initial study and subsequently completed the final version of the analysis. Following the second analysis, the data were reanalyzed by a field expert academician, and the two analyses were subsequently compared, and the percentage of consensus was calculated as 92% (Miles, 1994). The mathematical literacy levels of the groups were interpreted using descriptive analysis. The scores obtained throughout the process, as well as the competency scores from the competency areas, were calculated. The arithmetic mean of these scores was then used to establish the lower and upper ranges for the data sets (Büyüköztürk, Çokluk & Köklü, 2020).

As the research adopts three levels of interpretation for success—low, moderate, and high—and as the maximum score that groups can obtain in the mathematics competency categories each week is 18, the mean has been calculated as 6. A mathematical literacy score falling between 0 and 5 is regarded as low, a score between 6 and 11 as moderate, and a score between 12 and 18 as high. During the four weeks, as the highest possible score that five groups can attain in a specific competency area is 60 and the research utilizes three categories—low, moderate, and high, the mean value has been determined as 20. A score ranging from 0 to 19 is categorized as a low-level skill, while a score ranging from 20 to 39 is classified as a moderate-level skill. Lastly, a score ranging from 40 to 60 is categorized as a high-level skill.

Table 2 explains the mathematical competency levels developed by Turner et al. (2014) to demonstrate mathematical literacy skills. When assessing the competency areas mentioned earlier, if the groups' performance in these areas is at Level 0, they are awarded 0 points; at Level 1, they are awarded 1 point; at Level 2, they are awarded 2 points; and at Level 3, they are awarded 3 points. Below is a sample coding based on the solution of Group 1, one of the focus groups of the second week, regarding the Restaurant Problem:

In the problem, students are trying to help the manager of a hamburger restaurant. The task of the students is to find out the reasons why the customers who go to the restaurant choose this restaurant. For this purpose, the students were presented with a table with various categories and the customers voted on these categories.

When Group 1's skill of forming problem-solving strategies (PS) related to this problem was evaluated, it was seen that Group 1 adopted a multi-stage strategy in solving the problem. The group first found the total score of the five categories presented to them. While doing this, they summed the scores of each category separately. Secondly, they sorted the total scores from largest to smallest and finally determined the most important category. Therefore, the problem-solving strategies skill of the group was coded as 2 points.

After Group 1's reasoning skill (RS) for this problem was examined, it was observed that Group 1 formed a chain of inferences by following a multi-step argument presented to them. The group made inferences based on a scoring table with five different categories of ten customers. The first inference of the group was to find the most important category for each customer. The fact that there were differences in the most important categories of each customer led the group to make another inference over the table, and this time, the group preferred to calculate the total scores of the categories and followed a reasoning process based on sorting these scores. Therefore, the reasoning skill of the group was evaluated as 2 points.

When Group 1's representation skill (RES) related to this problem was analyzed, it was observed that Group 1 was able to read the values given on the table correctly, interpret and use the relationships among customers, categories, and scores, and make comparisons based on these values. Therefore, the representation skill of the group was coded as 2 points.

Table 2
Mathematical Competency Levels

Competency Levels	Level 0	Level 1	Level 2	Level 3
Representation Skill	The absence of any representation or a simple representation, such as reading values from a coordinate system or a table.	Employing a straightforward and conventional representation to interpret the relationships in the provided scenario. Similar to interpreting values from a graph and conducting comparisons.	To comprehend and make use of a complex representation, one must both finish and construct the representation, with some of the structure being supplied by the essential framework.	Understanding and utilizing multiple complex representations; establishing connections between representations, making comparisons, and evaluating them. Transformation of a complex mathematical expression into representation.
Communication Skill	The communication consists of short sentences that only include the presentation of a single word or numerical result.	Communication should be simple; like writing a short statement or calculation, or expressing a range of values.	A brief explanation in communication or the presentation of a series of calculation steps.	The presentation of arguments that connect multiple elements of a problem or solution in communication.
Reasoning Skill	Making direct inferences from the given information and instructions.	Utilizing the requisite procedures for logical thinking and making inferences from the given data in a basic mathematical issue.	Combining different aspects of the problem or identifying the complex relationships within the problem and drawing conclusions. Creating a chain of inferences to follow or construct a multi-step argument.	The use or creation of interconnected chains of inference. Controlling, justifying, and evaluating complex inferences and synthesizing them.
Problem Solving Strategies Skills	The necessary steps for a solution in the resolution process should be explicitly articulated and apparent.	Using a simple strategy that generally consists of a single step in solving problems.	Using a multi-step strategy in problem-solving.	Using a complex, multi-step strategy that involves bringing together multiple sub-goals or evaluating and comparing strategies.
Modelling Skills	The presented scenario is either purely mathematical or the relationship between the given circumstance and the established model is not relevant to solving the problem.	Creating a model that takes into account the assumptions, variables, and interactions in the provided scenario.	The process involves constructing a model by taking into account the assumptions, variables, and relationships in the given situation, making adjustments to a particular model, or analyzing and understanding a model or mathematical outcomes.	Constructing a model by taking into account the underlying assumptions, variables, and relationships in the given scenario, then validating or assessing the models in respect to the problem at hand, and finally comparing and establishing connections between different models.
Symbolic Skill	Performing short arithmetic calculations that only involve easily traceable numbers.	The ability to use arithmetic calculations that include fractions and decimal numbers, alongside basic arithmetic operations. It's like using the fact that the sum of the interior angles of a triangle is 180 degrees to find a missing angle.	The simultaneous use of expressions, rules, and formulas that have multiple components. It's like rearranging a formula algebraically.	The application of multi-step mathematical procedures that combine various rules, definitions, and techniques. It's like deciding which algebraic expression form would be better to use for a specific purpose.

With the evaluation of Group 1's communication skill (CS) related to this problem, it was observed that Group 1 provided both an explanation for the solution of the problem in the report paper after solving the problem and presented an argument linking multiple elements of the solution by drawing a graph and showing the order they presented in the solution. It was also noticed that Group 1 wrote the calculation steps they used while solving the problem in the report. Therefore, the communication skill of the group was evaluated as 3 points

When Group 1's modeling skill (MS) related to this problem was evaluated, it was observed that Group 1 created a model by considering the assumptions, variables, and relationships in the given problem situation and interpreted the mathematical results of this model. Group 1 reached the total score by adding each category. Then, it was found that Group 1 formed their models by ranking these scores from the highest to the lowest and determining the best category. Group 1 built its model on a three-stage system. Firstly, they read and interpreted the data from the table given to them, then they decided what to do and aimed to solve by using addition and sequencing. In the last step, they reached the solution by performing the mathematical operations they planned, and then they reported and explained it. Since Group 1 determined the variables and the relationships between them based on the problem situation, interpreted and created their models, the modeling skill of the group was coded as 2 points.

Considering the symbol skill (SS) of Group 1 for this problem, the symbol skill of Group 1 was coded as 0 (zero point) because the group only performed short arithmetic calculations (addition and sorting) with easily traceable numbers from 1 to 5.

4. Results

In this section, firstly, the solutions and reports generated by students for modeling activities have been assessed in the context of mathematical literacy, and specific examples of solutions have been provided. Secondly, the data collected from the students' dairies were examined and excerpts from their comments were presented to provide a comprehensive understanding of their views on the modeling process.

4.1. The Mathematical Literacy of Students in the Context of Mathematical Modeling

4.1.1. Modeling Activity for the First Week: The Holiday Problem

A holiday problem was incorporated during the first week of the modeling activities. In the holiday problem, students have attempted to identify the most appropriate cities for two consumers who submitted applications to the travel agency. They have employed a table that contains information about the cities, considering the preferences of the consumers, to identify these cities.

Table 3 summarizes the results of the students' solutions and reports concerning the holiday problem that was implemented during the initial week. The rating system assigns a value of 0 to the lowest level of competency, 3, to the highest level of competency, and X to the competency area that has not been observed.

Table 3
The Scores of Mathematical Literacy Skills in the First Week

Groups	*PS*	*RS*	*RES*	*CS*	*MS*	*SS*	*Total Score*
Group 1	2	2	1	2	2	0	9
Group 2	1	1	1	2	2	X	7
Group 3	2	2	1	3	2	X	10
Group 4	1	0	1	1	1	X	4
Group 5	1	0	2	0	1	X	4
Total	7	5	6	8	8	0	

Note. PS: Problem Solving Strategies Creation Skills; RS: Reasoning Skill; RES: Representation Skill; CS: Communucation Skill; MS: Modelling Skill; SS: Symbolic Skill.

When Table 3 is evaluated, it is evident that the only category in which full scores were attained is communication skills. Group 3, which achieved the highest level of performance compared to the other groups, also obtained a perfect score for their communication skills. Among the competency areas, *communication* and *modeling skills* had the highest scores, both achieving a total of 8 points. However, it is observed that there are differences in the competency areas related to the levels of mathematical literacy among the groups. Except for Group 1, no evidence of symbolic skill has been detected in the other groups. The symbolic skill solutions of this group have received a score of 0 due to their reliance only on simple arithmetic calculations. Hence, it is evident that the *symbolic skill* is the least exhibited skill in the modeling activity. It has been determined that Groups 1, 2, and 3 demonstrate a moderate level of skill when the maximum score (18) from the six competency levels related to mathematical literacy is taken into account. Conversely, Groups 4 and 5 demonstrate low mathematical literacy skills in light of this problem.

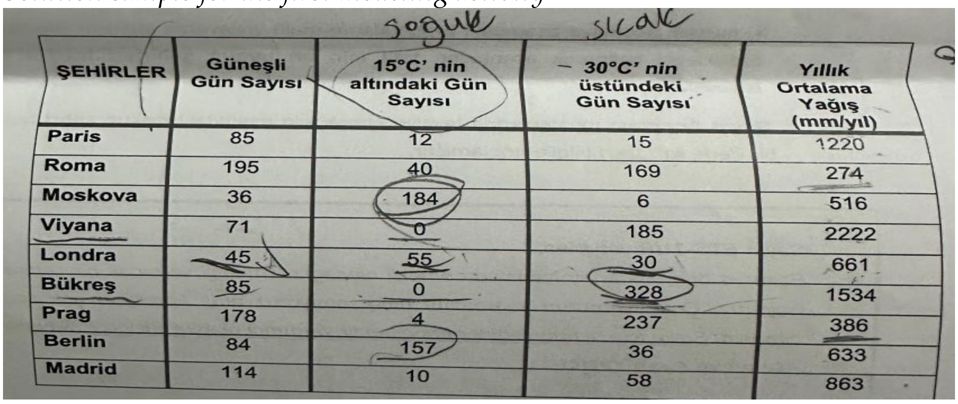
The following passages are selected from the solutions and report papers of the groups, and they include instances that illustrate the scoring process described in the table above. An illustration extracted from Group 3, showcasing a moderate level of mathematical literary proficiency and achieving a score of 2 in reasoning skill, is presented below:

> I believe Rome is the best spot for you because Rome has 195 sunny days. There have been 40 days with temperatures below 15 degrees. The total number of days with temperatures above 30 degrees is 169. The yearly average rainfall is 274 days. (Group 3)

For this level, which is scored as 2 for reasoning skill, Turner et al. (2014) assert that it is necessary to either identify the complex relationships within the problem and draw inferences or to combine the distinct aspects of the problem. Upon examination of the students' answers, it is evident that they consider and combine various elements in their responses, leading them to form a conclusion based on these variables. The expressions stated by the students on their report papers such as "number of sunny days", "the yearly average rainfall", "number of days below 15 degrees", and "number of days above 30 degrees" are all examples of variables. The students have deduced conclusions by taking into account all these variables, making connections between them, and presenting explanations. The provided response labeled as "Rome" is an outcome of this logical thought process. Due to this factor, the group's reasoning skill has been awarded as 2.

The following sample is extracted from Group 2, the constant focus group, and illustrates a moderate level of literacy abilities for this issue, scored as 1 for representation skill.

Figure 1
Solution sample for the first modeling activity



ŞEHİRLER	Güneşli Gün Sayısı	15°C' nin altındaki Gün Sayısı	30°C' nin üstündeki Gün Sayısı	Yıllık Ortalama Yağış (mm/yıl)
Paris	85	12	15	1220
Roma	195	40	169	274
Moskova	36	184	6	516
Viyana	71	0	185	2222
Londra	45	55	30	661
Bükreş	85	0	328	1534
Prag	178	4	237	386
Berlin	84	157	36	633
Madrid	114	10	58	863

According to Turner et al. (2014), at this level, which is given a score of 1 for representation skill, it is recommended to utilize a basic and conventional representation. This entails reading data from the graph and making comparisons. The excerpt above illustrates that students examined the values in the provided table, identified the highest values, and successfully obtained the required data through a comparative analysis. They derived a conclusion from the data based on the descriptors "cold" and "hot". How Group 2 made the notes and conclusions can be seen in the excerpts taken from their conversations:

Student D: Ahmet and Ayşe say it should be sunny.
Student E: I hope it is sunny, however...
Student Y: But they don't mind whether it rains.
Student E: They also don't want to be in a cold place.
Student D: The temperature is 15 degrees; it is accepted as hot.
Student E: 15 degrees is accepted as hot and 30 degrees as cold.
Student D: It should have been minus for that; 30 degrees is hot. It should have said -30 degrees. So they want this part: Moscow (marked the number of days with temperatures below 15 degrees on the line including Moscow).
Student E: But here, the number of days above 30 degrees is six.
Student D: wait a minute, 30 degrees is accepted as hot. Any temperature below 15 degrees is considered chilly. Let us write "cold" here. (The word "cold" is written in the appropriate region of the table. It was written "ho" for the days exceeding 30 degrees)
Student Y: So it is Bucharest then. They don't want it to be cold. (The number of days over 30 degrees was marked.)
Student D: Wait a minute; Student Y is correct because it is Bucharest, after all. Let us write Bucharest here. Wait a minute; I have an idea. Please read me the number of sunny days one at a time. Read me the number of sunny days, from highest to lowest. (They read the statistics about the cities from the table again.)
Student D: Prague has 178 sunny days, thus the answer can be Prague. Because there are four days with temperatures below 15 degrees, 237 days with temperatures over 30 degrees, and a yearly average rainfall of 386.
Student E: It will undoubtedly be Prague.
Student D: Let us write down the best possibilities. Let us write first, second, and third here... (Group 2)

Upon analyzing the discourse above, it becomes evident how the students employ their skills in representing ideas or concepts. The conversation commences with the students determining the specific requirements that clients have selected for their holiday. Further efforts were made to derive inferences from the facts presented in the table, wherein Student E asserted that 15 degrees is considered warm while 30 degrees is deemed cold. However, the initial conclusion drawn was inaccurate. Student D engaged in a discussion with a friend and countered his views while providing valid reasons to support his own perspective. The dialogue reveals that the students demonstrate their ability to represent information by doing tasks such as extracting data from a table and making relevant annotations (such as recognizing the highest value, etc.). During the ongoing debate, the students are observed engaging in comparisons between cities based on the facts displayed on the table, such as the number of sunny days and the yearly average rainfall. Comparing data is seen as another manifestation of representation skill

The following sample, taken from Group 5, illustrates low-level mathematical literacy skills and has received a score of 0 for communication skill in this problem.

-The cities suggested for the first client
"The cities that we suggest for you are: Rome, Madrid, Berlin, Prague, and Vienna.
-The cities suggested for the second client
The cities that we suggest for you are: Moscow, Vienna, London, Bucharest, and Berlin.
With all due respect, I present it to you." (Group 5)

Turner et al. (2014) state that at this level, which is given a score of 0 for communication skill, only short phrases containing a single word or numerical outcome should be used. When analyzing the answers of the students, it was noted that they offered concise and straightforward solutions by using brief, single-word terms such as Rome, Madrid, and Berlin when composing their reports on the solution. This answer lacks reason and fails to provide a clear explanation of the solution process to the client. As a result, the group's communication skill has been scored as 0.

4.1.2. Modeling Activity for the Second Week: The Restaurant Problem

During the second week of the modeling activities, the restaurant problem was put into practice. The students are endeavoring to assist the owner of the hamburger establishment in this particular scenario. The students are required to ascertain the factors that influence clients' selection of this eatery. To facilitate this, a table has been provided to the students, containing different criteria that customers can evaluate and vote on. The results derived from the students' solution papers and reports about this issue are presented in Table 4.

Table 4
The Scores of Mathematical Literacy Skills in the Second Week

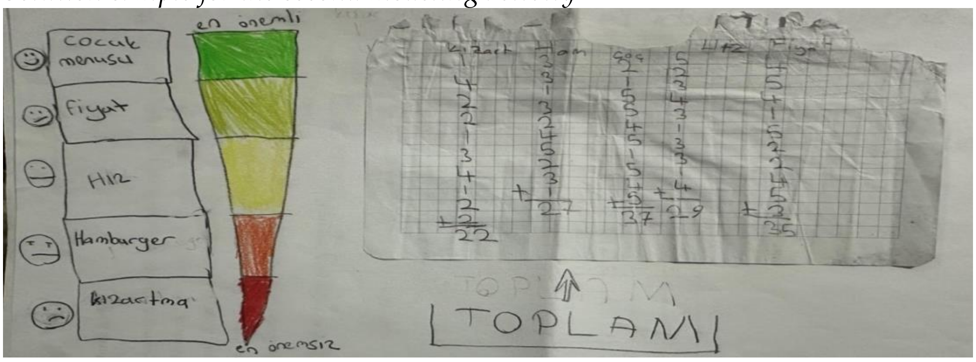
Groups	*PS*	*RS*	*RES*	*CS*	*MS*	*SS*	*Total Score*
Group 1	2	2	2	3	2	0	11
Group 2	2	2	2	3	2	0	11
Group 3	1	1	3	2	1	X	8
Group 4	2	2	2	2	2	0	10
Group 5	2	2	3	1	2	0	10
Total	9	9	12	11	9	0	

Note. PS: Problem Solving Strategies Creation Skills; RS: Reasoning Skill; RES: Representation Skill; CS: Communucation Skill; MS: Modelling Skill; SS: Symbolic Skill.

While evaluating the table above, it has been noted that there is a rise in the overall scores of all groups, except Group 3, in comparison to the previous week. Groups 1 and 2 have exhibited the highest results. Groups 1 and 2 obtained full points in the communication skills category, while Groups 3 and 5 received full points in the representation skill category. The skill with the highest score in the competency areas was *representation skill*, with a total of 12 points, followed by *communication skill* with a total of 11 points. Except for Group 3, it has been noted that the evaluation of symbolic skill has been carried out in the remaining groups, in contrast to the previous week. However, it is worth mentioning that the overall score achieved in the *symbolic skill* is significantly lower when compared to other skills and is the least frequently observed skill. After evaluating the six competency levels associated with mathematical literacy, it has been determined that Groups 1 and 2 display a high level of skill, Groups 4 and 5 demonstrate a moderate level of skill, and Group 3 exhibits a low level of skill in mathematical literacy concerning this problem.

The following sample, taken from Group 5, demonstrates a moderate level of mathematical literacy and has been assigned a score of 3 for representation skill in this problem.

Figure 2
Solution sample for the second modeling activity



According to Turner et al. (2014), at this level, which is rated as a 3 in terms of representation skill, it is crucial to comprehend and employ various intricate representations. Additionally, it is important to establish connections between these representations, make comparisons and evaluations, and convert a complex mathematical expression into a representation. The remark above highlights the students' process of calculating the total points for each category and subsequently arranging them in descending order. Following the students' creation of this rating, they utilized both facial expressions and colors to construct a visual representation. They have converted a quantitative rating into alternative visual representations utilizing facial expressions and colors. They utilized a smiling face and green color to symbolize the highest score, and a sad face and red color to indicate the category with the lowest score. The students have employed many representations in this solution, built connections between them, and arrived at a conclusion by comparing each circumstance with one another. Consequently, the representation skill has been evaluated as 3 points.

The following sample was taken from Group 1, one of the focus groups in the second week. The group demonstrates a high level of mathematical literacy and modeling skill was scored as 2.

> The menu of the children comes first since it scored the highest—37. Second place goes to the pricing since its score is 35. Speed with 29 points comes third. With a total score of 27 the hamburger came in fourth place. With a total of 22 points, fried food boasts the lowest score among all the foods. We ranked the hamburgers based on the meal scores we gathered and set them from most to least important.

According to Turner et al. (2014), at this level, which is scored as 2 in terms of modeling skill, it is essential to develop a model that considers the assumptions, variables, and relationships in the current circumstance. This can involve modifying an existing model or interpreting a model or mathematical results. As stated by Group 1 in the earlier remark, the overall score was obtained by adding together the values for each category. Subsequently, Group 1 is observed to arrange these scores in descending order, construct their models, and ascertain the optimal category. Group 1 has based its model on a three-stage system. Initially, the data was evaluated by extracting information from the provided table. Subsequently, a solution was sought by determining the appropriate course of action, employing addition and sorting techniques. They arrived at the solution by executing the predetermined mathematical procedures, and thereafter recorded and elucidated it in a report. Group 1 received a score of 2 for their modeling skills since they successfully recognized the variables and their correlations, understood them, and developed their models based on the presented problem circumstance. The conversations of Group 1 provided below indicate that they followed a system as mentioned above:

> Student F: Let's add them all together: four plus five equals nine, nine plus four equals thirteen... thirty-five.
>
> Student A: Let us write thirty-five here. (She points to the pricing column.)
>
> Student F: (begins to sum up the children's menu column). Five plus two equals seven; three more equals 10; twenty-four, twenty-nine, and thirty-seven.
>
> Student A: I believe that the children's menu is the best.
>
> Student T: They prioritize speed.
>
> Student F: (begins to add up the speed column). Four, eight,…… twenty-nine. Write twenty-nine here. (Continues by totaling the hamburger column). Sixteen, seventeen,…. twenty,….twenty-seven. Write twenty-seven here.
>
> Student T: They prioritize the hamburgers most.
>
> Student F: Wait a minute. (She begins to sum up the fried column.) One plus four plus five equals two more, seven, nine, 10, and twenty-two. Write 22 here as well.
>
> Student A: We'll now rank them from most important to least important.
>
> Student F: The most significant is thirty-seven.
>
> Student T: No, that is not it. Do not do it; you are making a mistake. They lay the greatest emphasis on frying because they scored it poorly. They assigned poor grades to what they valued the most.
>
> Student A: The first kids' menu is the best. We shall sort from largest to smallest.
>
> Student F: Teacher, could you please come here for a moment? (The teacher arrives.)
>
> Student F: Teacher, I'd want to say something. Which one is the most significant? The highest of all the scores?
>
> Teacher: Yes.
>
> Student F: (begins to organize the scores they wrote under the table) One, two, three, four, and five.
>
> Student A: The fry is the least important one.

Student F: At the moment, the first thing is most crucial. The children's meal is given the highest priority. Because they are the person who has the most points. The total score is 37. That's why this is the first. (Group 1)

Upon analyzing the discourse above, it is clear that the students initially aggregate the scores sequentially assigned to each category. After calculating the cumulative score for each category, it has become evident that there is a divergence of opinions among the group members on the order of ranking. It has been discussed among the group members whether the most important category is the one with the highest score or the one with the lowest score. It seems that the teacher is being sought to assist in resolving the dispute. Rather than participating in a deliberative process among them, students prefer to consult the teacher, whom they regard as a superior authority, at this point. This situation also offers insight into the teacher's role in the classroom and the students' habits during the lesson process. The students completed the mathematical modeling process by ranking the categories from most important to least important after receiving support from the teacher.

4.1.3. Modeling Activity for the Second Week: The Family Dinner Problem

The family dinner problem was resolved during the third week of the modeling activities. The students are tasked with assisting the three siblings in the preparation of supper for their parents in this problem. The students have been given a menu for meals. In addition to this menu, information about the number of steps and preparation time for each meal is provided for the students. Students have been instructed to create a work plan to enable siblings to cook a supper within four and a half hours. The results derived from the students' solutions and reports about this problem are presented in Table 5.

Table 5
The Scores of Mathematical Literacy Skills in the Third Week

Groups	*PS*	*RS*	*RES*	*CS*	*MS*	*SS*	*Total Score*
Group 1	0	0	1	1	1	1	4
Group 2	0	0	0	1	1	0	2
Group 3	2	2	1	2	1	1	9
Group 4	1	1	2	1	1	1	7
Group 5	2	2	1	1	1	0	7
Total	5	5	5	6	5	3	

Note. PS: Problem Solving Strategies Creation Skills; RS: Reasoning Skill; RES: Representation Skill; CS: Communucation Skill; MS: Modelling Skill; SS: Symbolic Skill.

When Table 5 is evaluated, it has been noted that there is a decrease in the total scores of all groups, except for Group 3, as compared to the previous week. Group 3 demonstrated the highest level of score, while Group 2 obtained the lowest overall score. It has been noted that the groups do not attain full scores in any area of competence. Group 3, which demonstrated the highest performance compared to the other groups, was awarded 2 points for problem-solving, reasoning, and communication skills, as well as 1 point for representation, modeling, and symbolic skills. When the scores in the competency areas are evaluated, it is evident that *communication skill* has the maximum score, totaling 6 points. A total of 5 points were obtained from all other competency areas, except symbolic skills, while a total of 3 points were obtained from the symbolic skills area. Similar to the previous two modeling exercises, the least demonstrated skill in this activity has been identified as *symbolic skill*. After evaluating the maximum score of 18 that may be achieved across the six competency levels in mathematical literacy, it has been determined that Groups 3, 4, and 5 exhibit a moderate level of mathematical literacy, whilst Groups 1 and 2 display low mathematical literacy in relation to this problem.

The following sample is an illustration from Group 1, demonstrating an example of low-level mathematical literacy skills and has been assigned a score of 0 for reasoning skill.

> Ece: She prepared rice pilaf, carrots, green beans, broccoli, fried potatoes, and zucchini. While doing these, Aylin prepares salad, dressing, bread, pavlova, a cheese tray, carrot sticks, and carrot dip. While Aylin is doing these things, Ali decorates the pavlova, cleans the dining table, organizes the room, lays out the tablecloth and napkins, sets the table, and cleans up after each meal. So, if the division of labor is completed, everything will be finished.

According to Turner et al. (2014), when reasoning skills are assessed as 0, it is necessary to make straightforward inferences based on the presented information and instructions. The quote above indicates that the pupils have drawn direct conclusions by utilizing the table given in conjunction with the problem scenario. It is seen that the students are incapable of providing a valid rationale for the allocation of responsibilities they have devised for the three siblings. The students failed to provide an explanation about the allocation of jobs and the utilization of mathematical operations or procedures during the allocation of tasks.

The following is an example from Group 3–one of the focus groups during the third week, which shows a moderate level of mathematical literacy and communication skill was scored as 2.

> -Ali; He'll prepare a salad with potatoes and fried zucchini. The other tasks include cleaning up after each meal and arranging the tablecloth and napkins. It takes 3 hours and 50 minutes to prepare these. Ali has 40 minutes left.
>
> -Aylin; In addition to decorating the Pavlova, the other duties include setting the table, tidying up after every meal, and organizing the space. These require two hours of preparation time. Aylin has two and a half hours left.
>
> -Ece; The plates of cheese, carrot sticks, and carrot dip. Other responsibilities include cleaning the dining room and tidying up after each meal. The preparation time is 2 hours and 5 minutes. The remaining time is 2 hours and 25 minutes.

At this level in which the communication skill was scored as 2, Turner et al. (2014) state that it is necessary to demonstrate a series of calculation steps or to provide concise explanations. When the provided quotation is examined, it is evident that the students successfully executed the intended allocation of tasks in problem circumstances and provided written explanations of this allocation of tasks. The students have evaluated the given meal list and task list collectively to identify the specific chores that the siblings will carry out. The duration of the siblings' work completion is measured, and the remaining time is also noted. The students have also considered the factor of time, instead of arbitrarily splitting the jobs. Group 3 has documented and explained the entire process in their papers, providing a concise summary. Due to this factor, the group's communication skill has received a rating of 2 points. In the modelling activity of Group 3, the conversation between them while reporting the tasks of Ali, one of the siblings, is given below.

> Student S: Yes, now we are starting to write. We are writing Ali's tasks.
>
> Student C: Ali; He will make potato and zucchini fries along with a salad. (Student K writes the explanations.)
>
> Student S: Write the other tasks.
>
> Student C: The other tasks include cleaning after each meal and laying out the tablecloth and napkins. (Student K continues to write the explanations.)
>
> Student C: We will now sum up the minutes of each of their duties.
>
> Student K: We are summing up. 20 minutes, 15, 30 then, 45, 30… it has not finished yet, they have also other tasks to be done. 30, 15…
>
> Student S: But wait, friends, we also need to convert four and a half hours into minutes. Because we did it by the minute.
>
> Student K: Okay. Let's convert minutes into hours. All of these add up to 3 hours and 50 minutes; what about the remaining time?
>
> Student S: We are subtracting three hours and fifty minutes from four and a half hours. We are also writing the results we found in Ali's section. (They perform the calculations altogether.)
>
> Student C: His remaining time is forty minutes…

The sample above illustrates that the students initially allocated the tasks among themselves. The students have assigned the duties to each sibling. Once these tasks were identified, the total duration of all the work was computed by executing an addition operation. The students transformed the obtained result from minutes to hours and then utilized subtraction to calculate the remaining time. The students have utilized the principle that “1 hour is equal to 60 minutes” by converting hours into minutes and performing a sequence of mathematical operations (addition, hour-minute conversion, division, and subtraction) systematically. As a consequence of this situation, the group's symbolic talent has been evaluated as 1 point.

4.1.4. Modeling Activity for the Fourth Week: The Airport Problem

The airport problem was implemented during the fourth week of the modeling activities. In this case, the students have attempted to determine the airline with the highest probability of punctual travel by analyzing the durations of delays among several airlines. The objective is to help the volleyball team that wants to travel abroad. The results obtained from the students’ solutions and reports on the problem implemented in the fourth week are presented in Table 6.

Table 6
The Scores of Mathematical Literacy Skills in the Fourth Week

Groups	*PS*	*RS*	*RES*	*CS*	*MS*	*SS*	*Total Score*
Group 1	2	1	2	2	2	0	9
Group 2	2	1	2	1	1	0	7
Group 3	2	1	2	1	2	0	8
Group 4	2	1	1	2	1	0	7
Group 5	2	1	1	1	1	0	6
Total	10	5	8	7	7	0	

Note. PS: Problem Solving Strategies Creation Skills; RS: Reasoning Skill; RES: Representation Skill; CS: Communucation Skill; MS: Modelling Skill; SS: Symbolic Skill.

When Table 6 is analyzed, it is evident that although the groups did not get full points in any competency areas, there is a general improvement in the total scores of the groups compared to the previous week. Although all the groups obtained a score of 0 for symbolic skills, they achieved a score of 1 for reasoning skills and 2 for creating problem-solving strategies. Group 1 has attained the highest level of performance compared to the other groups, accumulating a total of 9 points. When the scores for the competency areas are reviewed, it is evident that the skill with the highest score is the aptitude for devising *problem-solving strategies creation skill*, with a cumulative score of 10 points. Amongst the several skills noticed throughout this modeling task, the least frequently observed talent has been recognized as *symbolic skill*. After evaluating the maximum score of 18 that may be achieved across the six competency levels, it has been determined that all groups exhibit a moderate level of mathematical literacy.

The following example is taken from Group 2, which is the constant focus group in the research. This group exhibits a moderate level of mathematical literacy skills and has been assigned a score of 2 for problem-solving strategies creation skill.

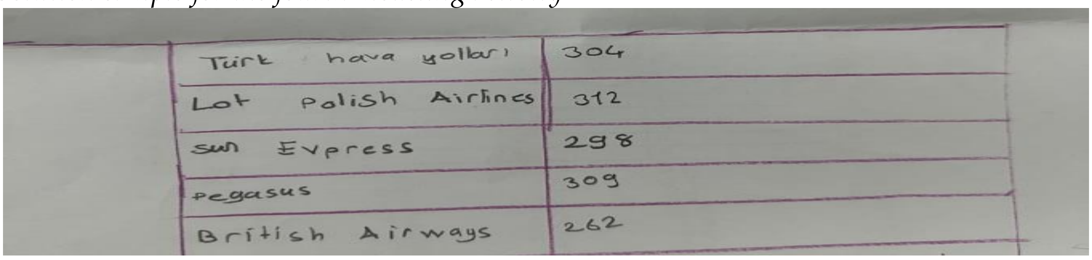
> Teacher: What are you doing at the moment?
>
> Student E: Teacher, (by pointing to the table) we added all of them. Then, we’ll put together all the airlines and determine how much they were delayed.
>
> Student Y: Let me have one as well. (Grabs the paper from a friend and does the addition). 25, 28, 36.... 134
>
> Student E: No 274
>
> Student Y: 134, come on, you make me feel confused. Which one was I going to add to 134?
>
> Student D: You were going to add 5. Let me add it. (She takes the paper and performs the addition operation). 309 Pegasus.
>
> Student E: (He takes the paper and makes the calculations for the next airline). 174, 178… 218… 227 no 234, 246… 261… 318, 323 (finishes calculating sunexpress then, makes calculations for lot polish airlines).

Student Y: You are not going to make all the calculations. (Then takes the paper from his friend and continues). That is all
Student E: Ok, let me do the writing. (Draws the table).
Student D: What are you doing?
Student E: Drawing a table. (Looks at the table). Which one is the highest?
Teacher: What are you doing right now?
Student E: Teacher, we have them ranked; but, have we yet determined how many minutes they are late?
Teacher: Yes.
Student E: Are we going to rank them from the best to worst?
Teacher: Yes.
Student: Ok then, let's rank them from the best to worst…

According to Turner et al. (2014), at this particular level, which was scored as 2 for problem-solving strategies creation skill, it is imperative to employ a multi-step method while solving problems. When solution strategies developed by students for this problem are analyzed, it comprises several sequential steps. The students were initially assessed by reading the given table. Next, they conducted the addition operation and recorded the resulting value in the designated column for each airline. Subsequently, they organized the accumulated figures and concluded the most appropriate airline company. Therefore, the solution steps for the method devised by the students consist of analyzing tables, executing addition operations, and generating sequences. From this standpoint, the students' skills to create problem-solving strategies have been evaluated as a score of 2 points. An important observation is that even when students are advancing correctly through the process and executing their mathematical operations accurately, they still rely on the teacher's advice and approval to verify the accuracy of the problem-solving strategies they employ.

The following sample was taken from Group 3, demonstrates a moderate level of mathematical literacy, and has received a score of 2 for modeling skill.

Figure 3
Solution sample for the fourth modeling activity



Türk hava yolları	304
Lot Polish Airlines	312
sun Evpress	298
pegasus	309
British Airways	262

According to Turner et al. (2014), at this level, which is scored as 2 for modeling skill, it is important to develop a model or make changes to an existing model by considering the assumptions, variables, and interactions in the given scenario. In this problem scenario, students have been given different variables, including the number of flights conducted by airlines and the duration of delays for these flights. Students have constructed their models by establishing relationships between these variables. Upon scrutinizing the students' solution papers, it is noted that they initially assessed the table provided to them by perusing the facts about the airline corporations. Subsequently, the delay periods for each airline company were estimated based on the delays provided in the table, leading to the determination of the delay duration. Ultimately, they sequentially arranged these time intervals from shortest to longest to construct their models. Finally, the students constructed a table to visually depict the findings they obtained from their modeling process regarding the top airline firm. Due to this reason, the group's modeling skill has been evaluated as 2 points.

The scores obtained by the groups over four weeks and their corresponding averages are displayed in Table 7.

Tablo 7
Week-based Scoring and Mean Scores

	Week 1	Week 2	Week 3	Week 4	Mean
Group 1	9	11	4	9	8.25
Group 2	7	11	2	7	6.75
Group 3	10	8	9	8	8.75
Group 4	4	10	7	7	7
Group 5	4	10	7	6	6.75

When Table 7 is analyzed, it is evident that Group 3 exhibits the highest performance, with an average score of 8.75. Group 1 achieved second place with an average score of 8.25, Group 4 secured third place with an average score of 7, while Groups 2 and 5 both obtained the same average score of 6.75, resulting in a tie for fourth place. When assessing the overall performance of the groups, it can be concluded that all groups demonstrated a moderate level of mathematical literacy skills, as the highest possible score was 18. Except for Group 3, all the other groups achieved their highest score in the second week, whereas Group 3 had its highest score in the first week. Although Groups 1 and 2 exhibited the lowest score during the third week, Groups 4 and 5 had their lowest score in the first week. Only Group 1 and Group 2 demonstrated a high level of mathematical literacy skills in the second week. It is evident that in previous weeks, the groups demonstrated mathematical literacy skills that may be assessed as moderate or low.

Table 8 shows the scores of the groups in mathematical literacy skill and their total scores for each skill for four weeks.

Tablo 8
Skill-based Scoring and Total Scores

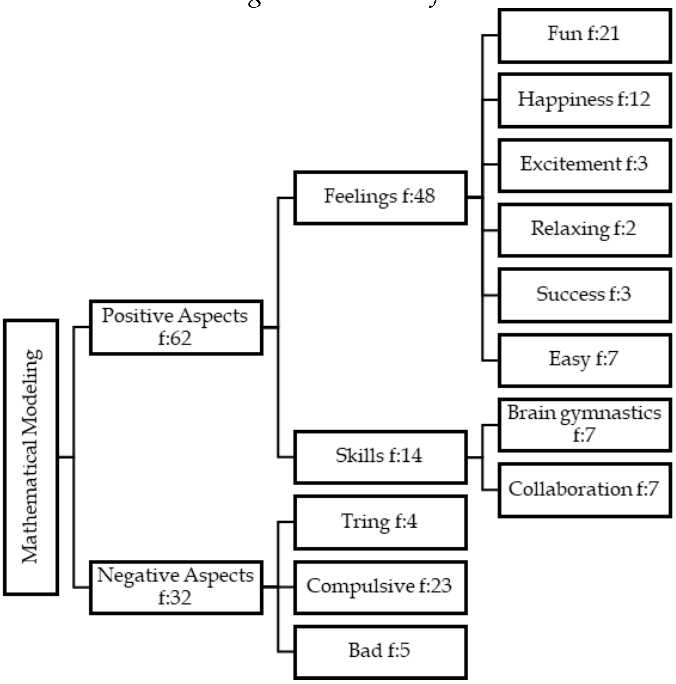
	Week 1	*Week 2*	*Week 3*	*Week 4*	*Total*
Problem Solving Strategies Creation Skills (PS)	7	9	5	10	31
Reasoning Skill (RS)	5	9	5	5	24
Representation Skill (RES)	6	12	5	8	31
Communucation Skill (CS)	8	11	6	7	32
Modelling Skill (MS)	8	9	5	7	29
Symbolic Skill (SS)	0	0	3	0	3

The scores obtained by the groups in each skill area of mathematical literacy, as well as their total scores after four weeks, are reported in Table 8. Over four weeks, the highest possible score that may be attained by each of the five groups in a particular skill area is 60. According to this, the students exhibit a moderate level of skill in all areas except for symbolic skill (SS). During all four modeling activities, students obtained a score of 32, which was the highest in the area of communication skills (CS), while they exhibited a score of 3, which was the lowest in the area of symbolic skills (SS). At the end of the process, the students achieved a score of 31 points in problem-solving strategies creation skills (PS) and representation skills (RES), 24 points in reasoning skills (RS), and 29 points in modeling skills (MS). When the students are analyzed based on all competency areas in terms of mathematical literacy, it is seen that the scores of other skills (PS, RS, RES, CS, and MS) except symbol skills are close to each other and students have these skills at a moderate level. It has been noted that the skills for which students received high or low scores varied during specific periods. For instance, students exhibited the highest level of score in the PS during the fourth week, while they exhibited the highest level of score in the RS, RES, CS, and MS during the second week. In the third week, students demonstrated the lowest level of score in PS, RES, CS, and MS. In contrast to the other skill areas, the highest score was obtained from the SS in the third week. From this standpoint, it is possible to assert that the content of modeling activities influences the skills that students exhibit, and they are better at displaying some skills and have difficulties to reflect others, depending on the specific modeling activity. For four weeks, the students generally exhibited a moderate level of all skills in mathematical literacy except for SS, even though their scores varied.

4.2. Students' Views on Mathematical Modeling

The content analysis was conducted on the diaries that students were required to maintain during the four weeks of mathematical modeling activities. The content analysis yielded the codes, categories, and themes depicted in Figure 4.

Figure 4
Themes and Code Categories obtained from Diaries



When the above diagram in Figure 4 is analyzed, it is evident that the views of the students were divided into two groups positive aspects (f:62) and negative aspects (f:32). The Positive aspects theme is divided into two categories. They are feelings (f:48) and skills (f:14) categories. The feelings category was coded under six headings fun (f:21), happiness (f:12), easy (f:7), excitement (f:3), success (f:3), and relaxing (f:2).

Students have evaluated mathematical modeling activities as fun, indicating that they had a wonderful time. The student coded as S10 stated her feelings and opinions as "I had a lot of fun in this class because I really enjoy collaborating with my friends." (1st week). The student coded as S9 reports that they experienced a high level of excitement, happiness, and enjoyment during the modeling activities: "I felt successful, happy, and excited. I am extremely grateful. We thought a lot and we succeeded..." (3rd Week). The student coded as S10 expressed, "I thoroughly enjoyed this assignment." I had a great time with my companions, and this was the quickest work we completed. I experienced achievement in this study..." (3rd week) by emphasizing that the process made her feel successful. The student coded as S12 found the mathematical modeling activities to be relaxing "Today was enjoyable and beautiful. We felt relieved when we saw that we could do it..." (2nd week). Mathematical modeling activities have been perceived as easy by certain students. One of these students, S11, articulates his views as follows: "I believe that today's activity was good and easy" (3rd week). The following was written in S5's diary with a similar expression: "The mathematical operations we completed today were easy. We promptly collected all of them and organized them. The results were recently released, however, it necessitated a significant amount of time" (4th week).

The titles of the skills category (f:13) are brain gymnastics (f:7) and collaboration (f:7). Students have indicated that they are engaging in a form of brain gymnastics by participating in mathematical modeling activities. It is noticeable that these activities stimulate their minds, as they record their experiences in their diaries. In relation to the subject, S10 stated that "... It was easy to solve the problem because my friends are very good listeners. We both had fun and were happy, and we all brainstormed, which opened our minds..." (2nd week). In the diaries they maintained during the mathematical modeling activities, a few students emphasized the importance of collaboration skills, emphasizing the importance of solidarity, cooperation, and mutual assistance. The opinions of S8 are as in the following on this topic: "Mutual assistance and solidarity are of paramount importance. Although it is exceedingly challenging, this is the method by which it is resolved. I comprehended this in this issue" (1st Week). In the same vein, S10 underscores the importance of collaboration by asserting, "...Even though the problem was challenging, we were able to resolve it in conjunction with our friends, although we encountered some challenges" (2nd week), emphasizing the significance of his companions' contributions to the solution of the problem.

Three codes were developed for the second theme to address the negative aspects of the modeling process. These codes are compulsive (f:23), tiring (f:4), and bad (f:5). The majority of students find mathematical modeling activities to be challenging when examining the negative aspects that were gathered from students' diaries. Students have disclosed that they encounter difficulties during mathematical modeling exercises. The diaries of students coded as S2, S7, and S12 are excerpted below.

S2 expressed her perceptions as "I believe it was challenging, but it was also enjoyable and went smoothly. We contemplated the matter extensively" (3rd week). S7 elaborated on the areas in which he encountered difficulty during the modeling activity in the first week as "I had a bit of difficulty because there were nine cities, we examined all the cities, and in the end, we found the results. We collaborated with all of my peers to compile our ideas and produce a solution" (1st week). Similarly, S12 underscored the difficult nature of the modeling activities by employing this expression "It was initially challenging to locate and configure. Of course, it was enjoyable as we did it, and as we continued, we started to find the results more easily..." (2nd week). In addition to this, she acknowledged that they became more adept at resolving them as time progressed.

When the student diaries were analyzed, it was noted that the modeling activities were deemed tiring by students in four separate instances. The student with code S12 has stated that the incessant writing is the cause of his fatigue, "It was somewhat challenging, but we eventually discovered it. I am feeling fatigued from writing and drawing, and my hands are aching. Nevertheless, I found it to be enjoyable and I am content" (4th week). A few students have disclosed that they experienced negative emotions during the modeling exercises. Because it is the initial week, two of these students are experiencing distress as a result of their disagreements with their colleagues. S2 stated "It was actually good; however, the atmosphere was unfavorable due to Student A. He is attributing the blame to me. I want him to leave the group" (1st week).

It was also determined that students conveyed their negative emotions during the modeling process using terms such as compulsive and tiring but these statements were exceedingly uncommon. Students predominantly highlighted the favorable features of the modeling process and conveyed their enjoyment and enthusiasm in participating in modeling activities, expressing a sense of happiness throughout the process. They prioritize highlighting the benefits of collaborating with friends in a group and engaging in brainstorming while modeling. It is determined that when students collaborate and generate ideas together, it enhances their problem-solving abilities in mathematical modeling activities. It became evident that although students have difficulties with modeling activities, they still find the overall process to be entertaining. Furthermore, it was determined that as they engaged in modeling activities, they had an increasing sense of comfort and proficiency, resulting in more efficient problem-solving.

5. Discussion and Conclusion

The fundamental aim of this study is to uncover the mathematical literacy skills of fourth-grade primary school students while engaging in mathematical modeling activities and their views on the modeling process. When the performance of groups in mathematical modeling activities and the skills related to mathematical literacy are analyzed, it is determined that all groups demonstrated a moderate level of mathematical literacy. Within the scope of mathematical literacy, it was observed that the students exhibited a moderate level of problem-solving strategies creation skill (PS), reasoning skill (RS), representation skill (RES), communication skill (CS), and modeling skill (MS) except for symbolic skill (SS). Upon examining the literature, it becomes evident that certain studies have found a low level of mathematical literacy among students, which contrasts with the present findings. These studies have utilized PISA questions and levels to evaluate mathematical literacy. Based on the findings of Purwanti et al. (2020), it can be inferred that the students' mathematical literacy skills are insufficiently developed and that their proficiency level in mathematics is at Level 1. When the findings of Kurniawati et al.'s (2020) study involving fourth-grade students were analyzed, it was noted that the students showed proficiency in answering Level 2 literacy questions but required more time to solve Level 3 questions. Diputra et al. (2019) found that fifth-grade students demonstrated proficiency in answering Level 1 questions as a means of evaluating their mathematical literacy. PISA notes that the minimum proficiency level required for students to participate in society in terms of mathematical literacy is Level 2. Students at this stage are developing the capacity to apply mathematical concepts in basic real-world scenarios. Students who fail to achieve this level demonstrate poor academic achievement (OECD, 2023). The students' low or moderate mathematical literacy, as observed in this study, can be attributed to various factors, including their limited exposure to modeling activities, their difficulty in applying mathematics to real-life situations, their consistent focus on routine problems, and consequently, their lack of familiarity with non-routine problems. Wigati et al. (2020) affirm the aforementioned perspective by asserting that students' deficient mathematical literacy skills result from an inability to answer practical math problems whereas Diputra et al. (2019) observed that students feel more comfortable when dealing with routine problems. When assessing this research, it is important to recognize that the participation of a primary school group in modeling exercises for the first time has resulted in a moderate degree of mathematical literacy. Because if students are given the opportunity, they have the potential to be raised as mathematically literate individuals from primary school onwards. According to the research conducted by Wei, Zhang, and Guo (2022), it is recommended that primary school students should be provided with the opportunity to engage in modeling activities; Firdaus and Herman (2017) emphasize that mathematical literacy should be developed from primary school age. Supporting primary school students in both mathematical modeling and mathematical literacy is a crucial matter that deserves significant emphasis.

Contrary to the existing research, this study suggests that the academic level of the study group can also be considered as a contributing factor to the students' moderate level of mathematical literacy skills. When assessing students' proficiency in *problem-solving strategies creation skills* (PS) to support this perspective, it is noted that they attain the highest ratings in this skill domain, along with representational skills (RS), after communication skills (CS). The study group consisted of a majority of high-achievement students, with only three students showing low achievement. This has given the students an advantage in devising problem-solving solutions. Saygılı (2017) reached a similar conclusion in his study with high school students, stating that there is a correlation between the students' achievement in mathematics and the problem-solving strategies they use. Students who possess high problem-solving skills employ a greater number of strategies, whereas students with low problem-solving skills employ fewer strategies and have encountered difficulties in solving some situations. Additionally, the study revealed that the problem-solving skills of the students involved in the research were generally assessed to be at a moderate level. Similarly, although high scores were obtained in this research compared to other skills, problem-solving strategies were generally assessed as being at a moderate level. Yayuk and As'ari (2020) conducted a study that explored the creative thinking skills of fifth-grade students, taking a different perspective on the topic. The researchers determined that students with low skills encountered obstacles when presented with non-routine situations and had difficulties in using problem-solving procedures. They have stated that students with high skills do not have difficulty solving math problems. Ensuring that students engage more with non-routine problems and modeling studies will provide opportunities for the development of this skill. At the same time, when students have the opportunity to engage in modeling activities, this will also enhance their success (Boaler, 2001; Pollak, 2003). According to Saygılı (2017), teaching students problem-solving strategies and providing them with more chances to solve non-routine problems can enhance their mathematical achievement and problem-solving skills. In their study, İncebacak and Ersoy (2016) assert that students exhibit greater proficiency in resolving problems that bear resemblance to ones they have previously encountered or resolved. The researchers emphasize that the majority of students face difficulties when attempting to solve non-routine problems, attributing this challenge to the prevalence of routine problems in the curriculum.

Additionally, the research reveals that students demonstrate a deficiency in symbolic skills (SS). The competencies within the skill area might be attributed as the cause for this. According to Turner et al. (2014), to obtain a perfect score of three points, one must engage in multi-step procedures utilizing a formula or rule and navigate intricate systems that encompass multiple factors. A score of zero indicates the presence of situations that entail basic arithmetic procedures. The symbolic skills expected to be used by students at the primary school level are often less complex. Thus, these skills might have shown a clustering in around zero scores. The chosen modeling activities might not have provided opportunities for the application of more advanced competencies in this skill area. Students attained the highest score in the third week in the domain of symbolic skills (SS). One possible explanation for this might be that the modeling activity is structured in a manner that facilitates the utilization of this skill.

Among the skills, students have achieved the highest scores in *communication skills* (CS). Subsequently, *representation skills* (RS) *and problem-solving strategies creation skills* (PS) have emerged. Students' higher scores in communication and representation skills, as opposed to other skills, can be linked to their active involvement in modeling activities. Modeling activities necessitate the utilization of communication and representation skills. In his study, English (2006) emphasizes the inherent communication necessity of modeling activities, stating that these activities lay the groundwork for children to develop their communication skills and allow them to use different representations to report their solutions. Mathematical modeling is already included as one of the skills covered in mathematical literacy. The research findings indicate that the students possess a moderate level of *modeling skills* (MS). Engaging in mathematical modeling exercises not only enhances students' abilities to communicate and depict ideas but also fosters their development of mathematical thinking and learning. Mathematical modeling enables students to acquire practical experience by utilizing their mathematical skills in real-life situations. Considering all these benefits, the importance of mathematical modeling becomes evident once again (Asempapa, 2015). Therefore, students who acquire expertise in this area will also contribute to the improvement of mathematical literacy. Nevertheless, students can apply the reasoning skills taught in mathematical literacy when developing their mathematical models, and they demonstrate many types of mathematical reasoning throughout the process of mathematical modeling. (Eric et al. 2016). The study found that students had a moderate level of proficiency in *reasoning skills* (RS) when engaging in the mathematical modeling process. The moderate level of students' modeling and reasoning skills implies a potential correlation between these two skills. According to Ambarita et al. (2018), students who are successful in mathematical modeling are more successful in problem-solving than those who have lower skills in modeling. Students lacking proficiency in modeling were unable to generate inquiries and, due to their inability to employ mathematical procedures, were unable to attain suitable outcomes. Therefore, it can be stated that engaging in mathematical modeling activities acts as a catalyst for enhancing mathematical reasoning. The mathematical modeling process uncovers important mathematical knowledge and concepts, including prediction, explanation, hypothesis formation, analysis, interpretation, comparison, and justification, among students (Eric, 2009).

When evaluating the scores that the groups received weekly, it can be seen that, with one exception, all other groups achieved their highest scores in the second week. Similarly, students demonstrated the highest scores in reasoning skills (RS), representation skills (RS), communication skills (CS), and modeling skills (MS) in the second week. This circumstance may be attributed to the fact that the modeling activity implemented in the second week was more straightforward than the activities conducted in the other weeks. In the analysis of the lowest scores, the first and third weeks are identified. During the third week, students demonstrated the lowest levels of proficiency in the following areas: problem-solving strategies creation skills (PS), representation skills (RS), communication skills (CS), and modeling skills (MS). The disadvantage of the first week has been experienced by groups that performed inadequately. Due to their inexperience with modeling activities, they are uncertain about the necessary steps to take. It is possible to infer that the groups that performed inadequately in the third week were influenced by the fact that the modeling activity implemented that week was the most challenging and intricate activity of the process. According to this viewpoint, the mathematical literacy skills of students are influenced by the degree of difficulty and simplicity of the modeling activities employed in the process. In their research, Ekawati, Susanti, and Chen (2020) assessed the mathematical literacy of students, concluding that most students are at a moderate and low level. They also observed that students thrived when they were tasked with solving simple problems.

This study examines the mathematics literacy levels of fourth-grade primary school students within the context of mathematical modeling, as well as their views on this process. In general, students found mathematical modeling activities to be entertaining. They reported that they enjoyed the activities and were content. The students have also reported that they experienced a sense of accomplishment and relaxation as they completed the modeling exercises. When the literature is examined, it is evident that there are studies that achieve comparable outcomes (Kaygısız & Şenel, 2022; Özdemir & Üzel, 2012; Watters, et al., 2004).

It is possible to assert that mathematical modeling activities provide students with emotional and social support, as well as facilitate their cognitive processes. It has been noted that students are also encouraged to exhibit values such as solidarity and mutual assistance, in addition to their ability to collaborate and brainstorm. There are studies in the literature that have obtained similar results. In their study, Wei et al. (2022) reported that mathematical modeling enables students to construct mathematical concepts and enhances the development of their metacognitive and communicative abilities. According to English (2006), students generate and develop their own mathematical ideas during modeling activities. The Asempapa (2015) modeling studies have been proposed as a means to enhance students' collaborative abilities and promote their mathematical thinking.

Modeling activities are generally perceived as challenging and demanding by some students. Several things contribute to this circumstance. Firstly, the modeling activities are extensive and time-consuming, requiring sequential processing and being cyclical. This drives students to continuously return to the beginning. A negative element associated with modeling activities is the challenges that students encounter when adjusting to the group they are collaborating with. During the initial stages of the process, particularly in the first and second weeks, there may be disputes and disagreements among group members until the group dynamics stabilize. Kaygız and Şenel (2022) reported similar findings in their research, asserting that modeling activities have beneficial effects on children's social skills. However, they cited challenges with group work and the lengthy duration of the modeling questions, which resulted in a lack of time. English (2003) took a different approach by asserting that students, through collaborative modeling exercises and engaging in debates, acquired the skills to resolve disagreements. From this standpoint, it may be contended that even an action we assess unfavorably in the modeling process can ultimately yield advantages for the student.

6. Limitations and Implications

The study found that fourth-grade students initially encountered challenges when engaging in mathematical modeling activities. However, over time, they successfully adjusted to the process and became more proficient in finding solutions. From this juncture, it can be explored through several studies whether modeling activities can be introduced at lower grade levels. In this study, it was observed that the content of modeling activities shaped the skills exhibited by the students. Depending on the modeling activities, some skills were exhibited more frequently and better, while some skills were more difficult to observe. In particular, it was seen that the modeling activities selected in this study were not sufficiently appropriate to reveal students' symbol skills. More specific studies using modeling activities for mathematical competence areas can be conducted. Considering the importance of all the skills addressed within the scope of mathematical literacy in terms of mathematics, qualitative studies can be conducted in which each of these competence areas is examined separately in depth. Another goal of this study was to reveal students' views on the modeling process, and it was found that students were supported both socially and cognitively in this process. Modeling activities can be used with an interdisciplinary approach to examine students' development of values, tendencies, and skills other than mathematics. This study was formulated based on the qualitative paradigm and carried out in a private school, where the mathematical literacy of the students was determined to be at a moderate level. Research can be undertaken to examine whether the climate of private schools, the socioeconomic status of families, and the intensity of lessons influence this condition. It is recommended to conduct similar studies in public schools and to perform quantitative studies comparing public and private schools.

Acknowledgements: This article has been derived from the doctoral thesis of the first author under the supervision of second and third authors.

Availability of the data: The corresponding author can be contacted for additional information. The article and other resources include the original contributions to the study.

Data availability: The dataset of the study is available from the corresponding author on reasonable request.

Declaration of interest: No conflict of interest is declared by author.

Ethics statement: Authors declared that the study was approved by İstanbul University-Cerrahpaşa Social Sciences and Humanities Research Ethics Committee on 29 December 2023 with approval code 861550.

Funding: The author received no financial support to conduct this article.

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